The design of Demand-Adaptive public transportation Systems: Meta-Schedules

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Outline

1. Demand Responsive transit Systems (e.g., DAR)
2. Demand Adaptive transit Systems (DAS)
3. Scheduling issues in DAS, DAR, and Traditional transit
4. The Meta-Schedule problem
   - single segment subproblem - sampling-based technique
   - sewing segments together
5. Conclusions
Flexibility in Transit Systems

- Offer competitive transportation services
  - Capture additional demand
  - Better address population needs
  - Cover larger/additional areas

- Sustainability
  - Reduce operating costs
  - Increase resource utilization

- Integration with traditional transportation systems
  - User point of view
  - Management point of view
Dial a Ride Systems

- Users ask for personalized rides (door-to-door service)
- But are served collectively
  - Similar to a collective taxi service
- Initially devised to meet needs of users with reduced mobility
- Extended (somewhat) to deal with “low demand” areas or periods
- Quite expensive compared to regular service
- Frequency? Regularity?
Demand Adaptive System (DAS)

- Combine DAR flexibility to traditional system regularity and low-cost
  - Compulsory stops with time windows
    - regular line
  - Optional stops on request (active users)
  - Segments: Set of optional stops between two consecutive compulsory stops
  - Users can still access the service at compulsory stops (passive users)
A DAS line

Passive users only
A DAS line

Active and passive users
DAS as feeder lines
Multiple Lines

Transfer points among flexible (and traditional) lines (compulsory stops)
Traditional Transit
- Passing time at each stop of the line
- Designed for medium-term periods (six months, one year)
- Users plan their trips based on published schedules
- Tactical planning decision (line definition: higher level)

DAR
- Particular to each vehicle tour
- Varying according to the actual requests
- Operational planning activity
Scheduling Issues for DAS

- Combines planning phases of traditional transit and DAR
- Two schedules are built
- **Meta-Schedule**: Similar to Traditional Transit
  - Basic line definition: time windows at compulsory stops
  - Users plan trips based on the published Meta-Schedule
  - Tactical planning decision (compulsory stops, segments, frequencies at higher level)
- Each departure schedule: Similar to DAR
  - Defines the actual vehicle itinerary
  - Varies according to actual requests
  - Must be compatible with the Master Schedule
  - Operational planning activity
Basic DAS Design Problem
Basic DAS Design Problem

Input

- Region to serve
Basic DAS Design Problem

Input

- Region to serve
- Demand

Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions
Basic DAS Design Problem

Input
- Region to serve
- Demand

Decisions
- Compulsory stops
Basic DAS Design Problem

Input
- Region to serve
- Demand

Decisions
- Compulsory stops
- Sequencing
Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

- Compulsory stops
- Sequencing
- Segments
Basic DAS Design Problem

Input
- Region to serve
- Demand

Decisions
- Compulsory stops
- Sequencing
- Segments
- Time windows
Basic DAS Design Problem

**Input**
- Region to serve
- Demand

**Decisions**
- Compulsory stops
- Sequencing
- Segments
- Time windows

**Goals**
- Low routing costs
- High level of service
Time Windows
Time Windows

- Policy
- Given a compulsory stop and its time window $[a, b]$
- The Vehicle
  - Must leave the compulsory stop within the time interval $[a, b]$
  - May arrive before $a$
Consequences

- Passive users must be at the compulsory stop not later than $a$
- Passengers on the bus may experience idle times
- Time windows represent bounds on user travel time
Time Windows

Main Features

- Distance

- Width
Main Features

- Distance
  - Time to serve the optional stop
  - Idle times at compulsory stops
- Width
Main Features

- Distance
  - Time to serve the optional stop
  - Idle times at compulsory stops

- Width
  - Flexibility
  - Long waiting times for passive users

Meta-Schedule: problem description

**Input**
- Topological design
- Demand for transportation
- Policy for time windows
Meta-Schedule: problem description

Input
- Topological design
- Demand for transportation
- Policy for time windows

Output
- A time window for each compulsory stop
Meta-Schedule: problem description

Input
- Topological design
- Demand for transportation
- Policy for time windows

Output
- A time window for each compulsory stop

Conflicting goals
- Provide sufficient time to serve optional stops
- Minimize total maximum time
- Small time windows
- Minimize *idle* times
- ....
Problem Setting and Assumptions
Problem Setting and Assumptions

Demand

- Probability distributions of requests for $o/d$ pairs
- $\Rightarrow$ Probability of at least one request derived for each optional stop
- $\Rightarrow$ Serve the set of requests $\Leftrightarrow$ Serve the set of requested stops
Problem Setting and Assumptions

- **Demand**
  - Probability distributions of requests for o/d pairs
  - ⇒ Probability of at least one request derived for each optional stop
  - ⇒ Serve the set of requests ⇔ Serve the set of requested stops

- **Time windows**
  - We consider time windows with common and fixed width $\delta$
  - Two possible easy extensions
    - Fixed but different width
    - Maximum width
Formaly

Input

- A sequence of compulsory stops
- A set of optional stops partitioned into segments
- Travel time $c_{ij}$ for the pair of stops $(i, j)$ in a segment
- Probability $p_i$ of being requested for service for optional stop $i$
Formaly

**Input**
- A sequence of compulsory stops
- A set of optional stops partitioned into *segments*
- Travel time $c_{ij}$ for the pair of stops $\langle i, j \rangle$ in a segment
- Probability $p_i$ of being requested for service for optional stop $i$

**Output**
- Time windows $[a_h, b_h]$ for compulsory stop $f_h$, $b_h - a_h = \delta$
- It reduces to finding a sequence $\{b_0, b_1, \ldots, b_n\}$
Formaly

- **Input**
  - A sequence of compulsory stops
  - A set of optional stops partitioned into *segments*
  - Travel time $c_{ij}$ for the pair of stops $(i, j)$ in a segment
  - Probability $p_i$ of being requested for service for optional stop $i$

- **Output**
  - Time windows $[a_h, b_h]$ for compulsory stop $f_h$, $b_h - a_h = \delta$
  - It reduces to finding a sequence $\{b_0, b_1, \ldots, b_n\}$

- **Goals**
  - Serve *any* requested optional stop with probability $P_\epsilon$
  - Minimize $b_n
Subproblem: Single Segment

- Input
- A segment
Subproblem: Single Segment

- Input
- A segment
- Probability $p_i$ for optional stop $i$ of being active
Subproblem: Single Segment

- Input
  - A segment
  - Probability $p_i$ for optional stop $i$ of being active
  - The departure time $\bar{L}_{h-1}$ at compulsory $f_{h-1}$
To any subset $S_h \in N_h$ of optional stops is associated

- Probability $p_{S_h}$ of being active
To any subset $S_h \in N_h$ of optional stops is associated
- Probability $p_{S_h}$ of being active
- Service Time $H_h \rightarrow$ (active set) Hamiltonian Path?
- Arrival Time $T_h$ at the second compulsory $f_h$
Subproblem: Single Segment

To any subset $S_h \in N_h$ of optional stops is associated
- Probability $p_{S_h}$ of being active
- Service Time $H_h \rightarrow$ (active set) Hamiltonian Path?
- Arrival Time $T_h$ at the second compulsory $f_h$

$H_h$ and $T_h$ are random variables
Subproblem: Single Segment

\[
T_h(\omega) = H_h(\omega) + \bar{L}_{h-1}
\]

\[
S_h(\omega)
\]

Goal
Subproblem: Single Segment

\[
T_h(\omega) = H_h(\omega) + \bar{L}_{h-1}
\]

- **Goal**
- Find the smallest value \(b_h\) such that \(b_h \geq T_h(\omega)\) with probability \(1 - \epsilon\) (to guarantee good service)
Goal

Find the smallest value $b_h$ such that

$b_h \geq T_h(\omega)$ with probability $1 - \epsilon$ (to guarantee good service)

We take $b_h = T_h^{1-\epsilon} = H_h^{1-\epsilon} + L_{h-1}$, with $T_h^{1-\epsilon}, H_h^{1-\epsilon}$ defined as

$\mathcal{P}\{H_h(\omega) \leq H_h^{1-\epsilon}\} \geq 1 - \epsilon$ and $\mathcal{P}\{T_h(\omega) \leq T_h^{1-\epsilon}\} \geq 1 - \epsilon$
Subproblem: Single Segment

\[ T_h(\omega) = H_h(\omega) + \bar{L}_{h-1} \]

Crucial Point
Subproblem: Single Segment

\[
T_h(\omega) = H_h(\omega) + \bar{L}_{h-1}
\]

Crucial Point

- How do we compute \( T_h^{1-\epsilon} \) or \( H_h^{1-\epsilon} \) (they differ by a constant)?
Subproblem: Single Segment

\[ \bar{L}_{h-1} \quad S_h(\omega) \quad \bar{L}_{h-1} \]

\[ f_{h-1} \quad \bar{L}_{h-1} \quad S_h(\omega) \quad f_h \quad p_i \]

\[ T_h(\omega) = H_h(\omega) + \bar{L}_{h-1} \]

**Crucial Point**

- How do we compute \( T_h^{1-\epsilon} \) or \( H_h^{1-\epsilon} \) (they differ by a constant)?

- Cumulative Distribution Function (CDF) and Probability Mass Function (PMF) for \( H_h(\omega) \) and \( T_h(\omega) \) are needed
Subproblem: Single Segment

\[ T_h(\omega) = H_h(\omega) + \bar{L}_{h-1} \]

Crucial Point

How do we compute \( T_h^{1-\epsilon} \) or \( H_h^{1-\epsilon} \) (they differ by a constant)?

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Requires \( 2^{|N_h|} \) service-time evaluations!
Subproblem: Single Segment

Crucial Point

How do we compute $T_{h}^{1-\epsilon}$ or $H_{h}^{1-\epsilon}$ (they differ by a constant)?

Cumulative Distribution Function (CDF) and Probability Mass Function (PMF) for $H_{h}(\omega)$ and $T_{h}(\omega)$ are needed

Requires $2^{|N_{h}|}$ service-time evaluations!

We estimate the PMF by sampling
Sampling Algorithm

- Take \( \{r_1, r_2, \ldots, r_l\} \) random samples of relatively small cardinality
- Compute the \( PMF_k, CDF_k \), and \( b_h^k \) for each sample \( r_k \)
- Compute the mean value and standard deviation of \( b_h^k \)
- If the standard deviation is small, i.e., the solution is *precise*, stop
- Otherwise, increment the cardinality of the samples and iterate
Sampling Algorithm

- Take \( \{r_1, r_2, \ldots, r_l\} \) random samples of relatively small cardinality
- Compute the \( PMF_k, CDF_k \), and \( b^k_h \) for each sample \( r_k \)
- Compute the mean value and standard deviation of \( b^k_h \)
- If the standard deviation is small, i.e., the solution is *precise*, stop
- Otherwise, increment the cardinality of the samples and iterate

Observations
- The algorithm is very simple. But
- No guarantee of unbiased solution
- The dimension of the sample may become critical
Experimentation

- Focus on the estimation of the service time for a single segment
- Square-shaped segments, side length 300
- Compulsory stops at the extremities of one diagonal
- Optional stops are uniformly distributed on the square
- To each optional stop is associated a positive probability, 50% in average
  \[ P_\varepsilon = 0.95 \]
- Hamiltonian Path computed by our basic B&C code
## Results A20

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Longest 1-Path | Hamiltonian Path | “Real” $\bar{b}_h(\varepsilon)$ |
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<td>558.7</td>
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Conclusions (1)

- The algorithm for estimating time windows is fast
- The number of samplings is not very important
- The cardinality of samples is important. Around 200
- Method successful in solving problems with high dimension
Sewing Segments

- We introduce a new random variable
  - The vehicle departure time $L_h(\omega)$ at compulsory stop $f_h$ where
    
    $a_h \leq L_h(\omega) \leq b_h$
  
    $T_h(\omega) = L_{h-1}(\omega) + H_h(\omega)$ needed to compute $[a_h, b_h]$.
  
- $T_h(\omega)$ and $L_h(\omega)$ are linked by the relation

\[
L_h(\omega) = \begin{cases} 
    T_h(\omega) & \text{if } \omega \mid a_h \leq T_h(\omega) \leq b_h; \\
    a_h & \text{if } \omega \mid T_h(\omega) < a_h; \\
    b_h & \text{if } \omega \mid T_h(\omega) > b_h. 
\end{cases}
\]

- Observe that $L_{h-1}(\omega)$ and $H_h(\omega)$ are independent.

- $T_h(\omega)$ can be computed by convoluting $L_{h-1}(\omega)$ and $H_h(\omega)$. 

Complete Scheme

Input

- Sequence of segments $G = \bigcup_{1,2,\ldots,n} G_h$
- Service probability $P_\epsilon = (1 - \epsilon)^n$
Complete Scheme

Input
- Sequence of segments $G = \bigcup_{1,2,\ldots,n} G_h$
- Service probability $P_\epsilon = (1 - \epsilon)^n$

Algorithm
1. For each segment $G_h$, $h \in \{1, 2, \ldots, n\}$
   (a) Compute PMF and CDF of $L_{h-1}(\omega)$
   (b) Compute PMF and CDF of $H_h(\omega)$
   (c) Compute PMF and CDF of $T_h(\omega)$ as the convolution of the PMFs of $L_{h-1}$ and $H_h$
   (d) Compute $T_h^{1-\epsilon}$ and set $b_h = T_h^{1-\epsilon}$. 
Complete Scheme

Input

- Sequence of segments $G = \bigcup_{1, 2, \ldots, n} G_h$
- Service probability $P_\epsilon = (1 - \epsilon)^n$

Algorithm

1. For each segment $G_h$, $h \in \{1, 2, \ldots, n\}$
   (a) Compute PMF and CDF of $L_{h-1}^1(\omega)$
   (b) Compute PMF and CDF of $H_h(\omega)$
   (c) Compute PMF and CDF of $T_h(\omega)$ as the convolution of the PMFs of $L_{h-1}$ and $H_h$
   (d) Compute $T_h^{1-\epsilon}$ and set $b_h = T_h^{1-\epsilon}$.

Output

- The best sequence $\{b_1, b_2, \ldots, b_n\}$
- Such that any randomly requested optional stop is served with probability $P_\epsilon$
Experimentation

- **Goal**
  - evaluate the quality of the produced master schedules

- **Given several:**
  - topological DAS line design
  - origin destination demand distributions

- **Compute several master schedules varying:**
  - time window width $\delta$
  - probabilities $\epsilon$

- **Simulate** operations monitoring:
  - percentage of rejected requests
  - passive user waiting times
  - idle times at compulsory
  - ...
## Results

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<tr>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$b_4$</th>
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<th>stdv(R/T)</th>
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Master schedule evaluated on 4-segment line, low and high demand, 100 realizations
Conclusions (2)

- Demand Adaptive transit Systems
- DAS scheduling different from DAR and traditional transit
- The Meta-Schedule problem
- May be efficiently addressed by sampling (single segment)
- Results for general framework are satisfactory under different demand and line scenarios