The design of Demand-Adaptive public transportation Systems: Meta-Schedules

Gabriel Teodor Crainic

Fausto Errico

ESG, UQAM and CIRRELT , Montreal

Federico Malucelli

DEI, Politecnico di Milano

Maddalena Nonato

Università di Ferrara

JOURNE SCIENTIFIQUE

Chaire de recherche industrielle CRSNG en management logistique ESG UQAM

Outline

1. Demand Responsive transit Systems (e.g., DAR)
2. Demand Adaptive transit Systems (DAS)
3. Scheduling issues in DAS, DAR, and Traditional transit
4. The Meta-Schedule problem
    - single segment subproblem - sampling-based technique
    - sewing segments together
5. Conclusions

Flexibility in Transit Systems

- Offer competitive transportation services
  - Capture additional demand
  - Better address population needs
  - Cover larger/additional areas
- Sustainability
  - Reduce operating costs
  - Increase resource utilization
- Integration with traditional transportation systems
  - User point of view
  - Management point of view

Dial a Ride Systems

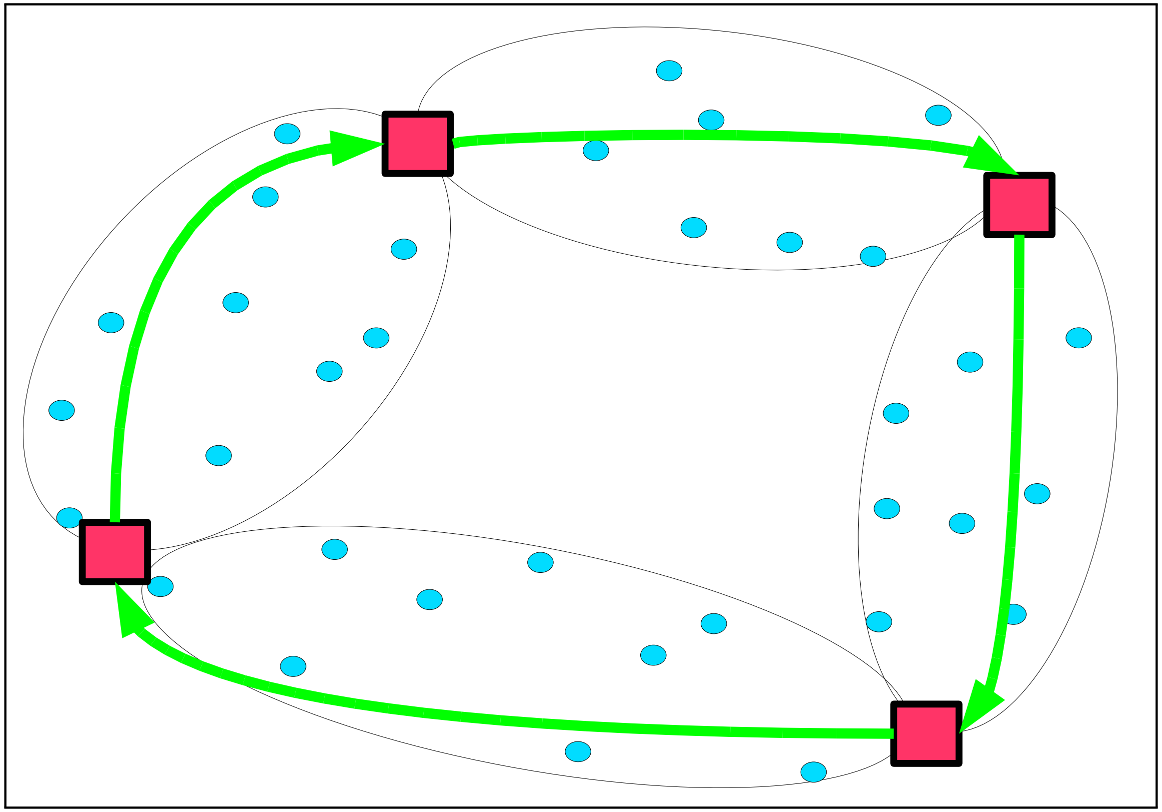
- Users ask for personalized rides (door-to-door service)
- But are served collectively
  - Similar to a collective taxi service
- Initially devised to meet needs of users with reduced mobility
- Extended (somewhat) to deal with “low demand” areas or periods
- Quite expensive compared to regular service
- Frequency? Regularity?

Demand Adaptive System (DAS)

- Combine DAR flexibility to traditional system regularity and *low*-cost
  - Compulsory stops with time windows
    - regular line
  - Optional stops on request (active users)
  - Segments: Set of optional stops between two consecutive compulsory stops
  - Users can still access the service at compulsory stops (passive users)

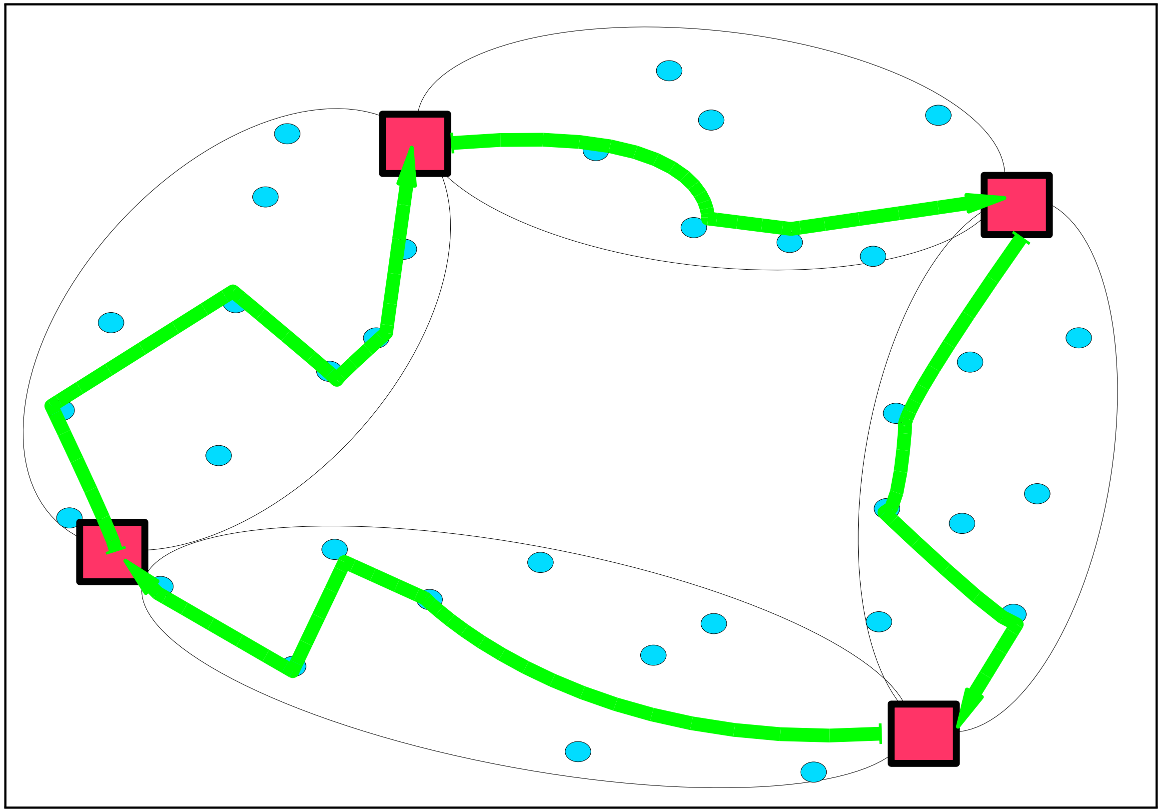
A DAS line

Passive users only

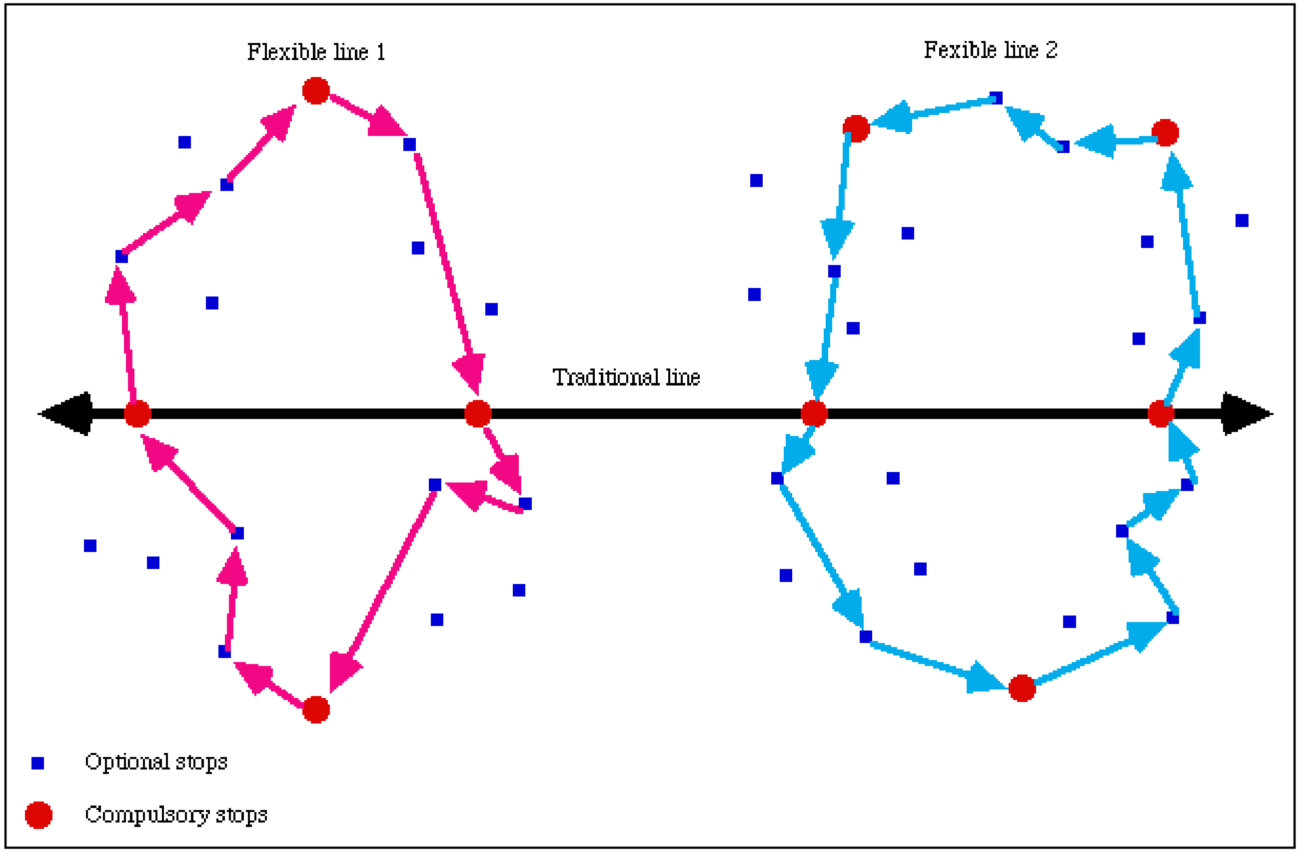


A DAS line

Active and passive users

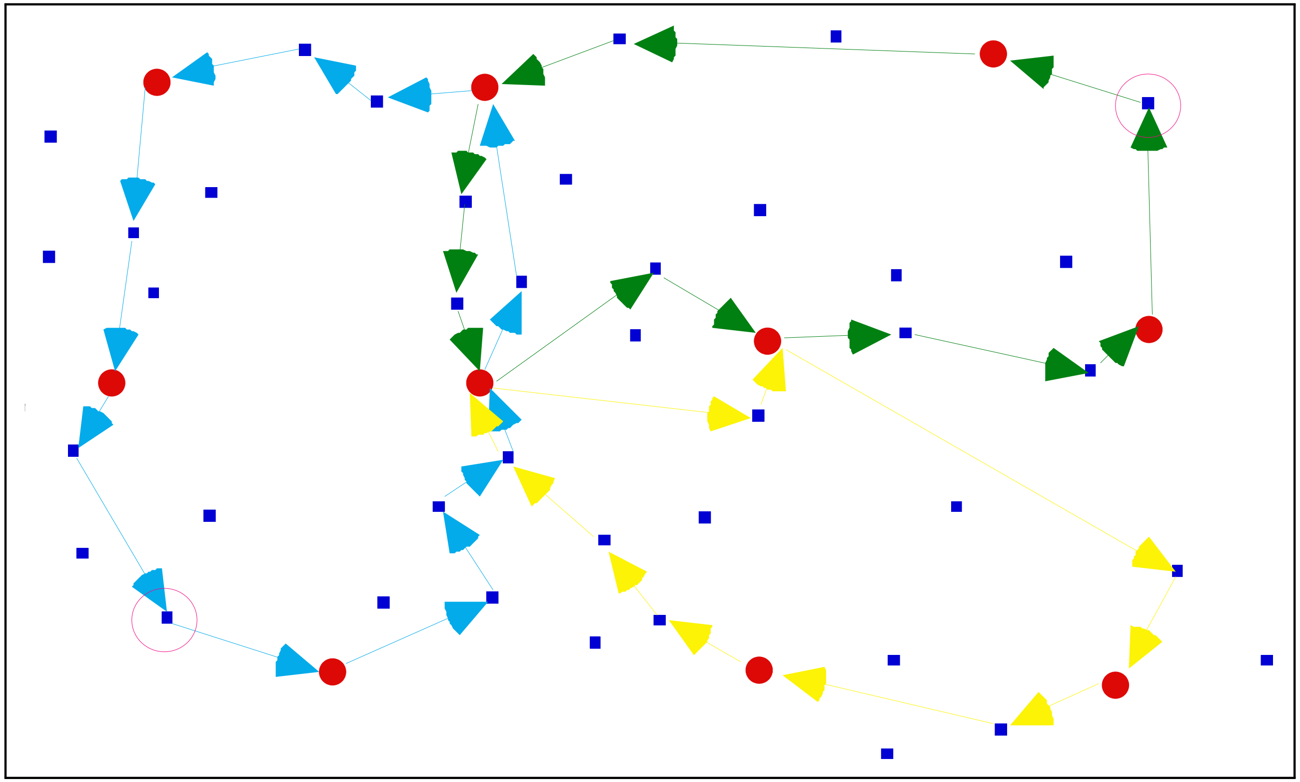


Integrated System



DAS as feeder lines

Multiple Lines



Transfer points among flexible (and traditional) lines (compulsory stops)

Schedules: Traditional Transit and DAR

- Traditional Transit
  - Passing time at each stop of the line
  - Designed for medium-term periods (six months, one year)
  - Users plan their trips based on published schedules
  - Tactical planning decision (line definition: higher level)
- DAR
  - Particular to each vehicle tour
  - Varying according to the actual requests
  - Operational planning activity

Scheduling Issues for DAS

- Combines planning phases of traditional transit and DAR
- Two schedules are built
- Meta-Schedule: Similar to Traditional Transit
  - Basic line definition: time windows at compulsory stops
  - Users plan trips based on the published Meta-Schedule
  - Tactical planning decision (compulsory stops, segments, frequencies at higher level)
- Each departure schedule: Similar to DAR
  - Defines the actual vehicle itinerary
  - Varies according to actual requests
  - Must be compatible with the Master Schedule
  - Operational planning activity

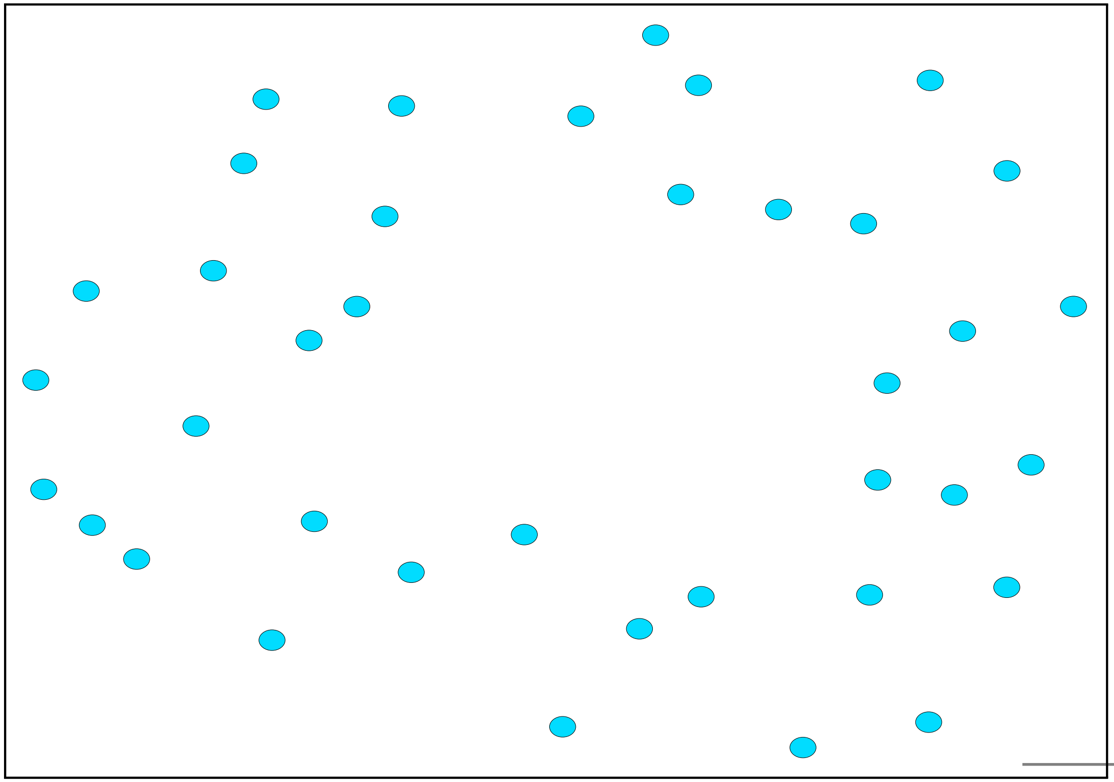
Basic DAS Design Problem

Input

Basic DAS Design Problem

Input

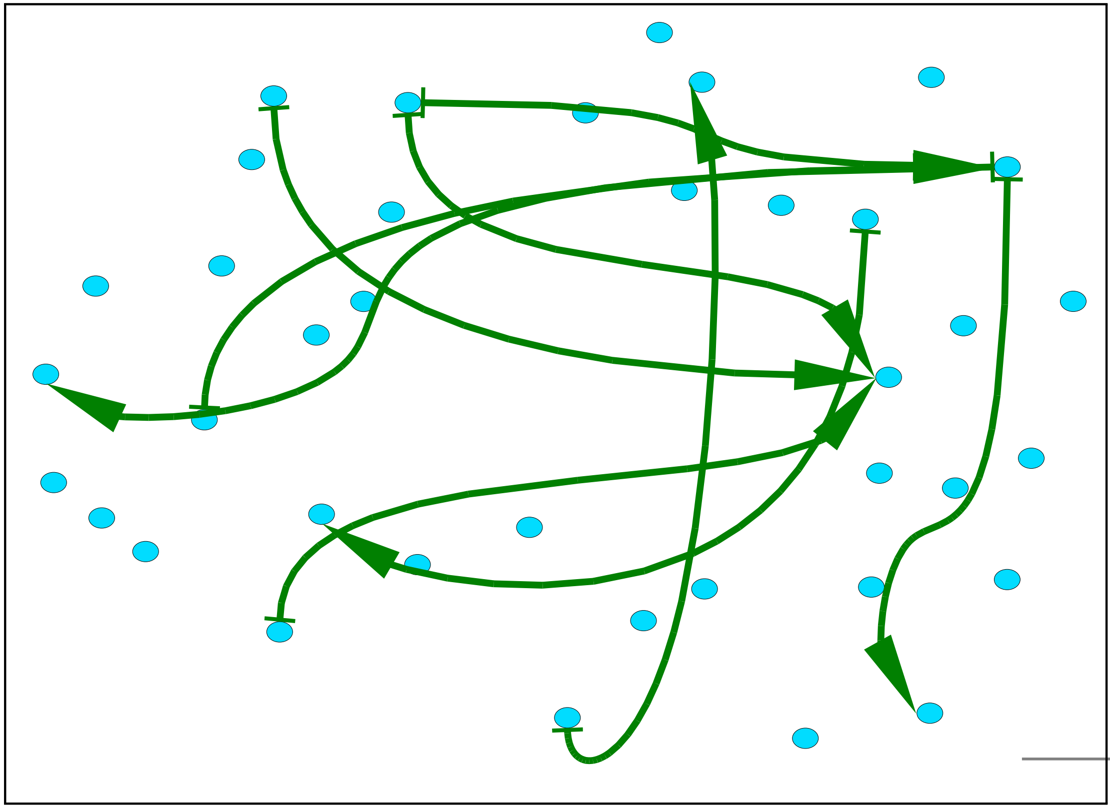
- Region to serve



Basic DAS Design Problem

Input

- Region to serve
- Demand



Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

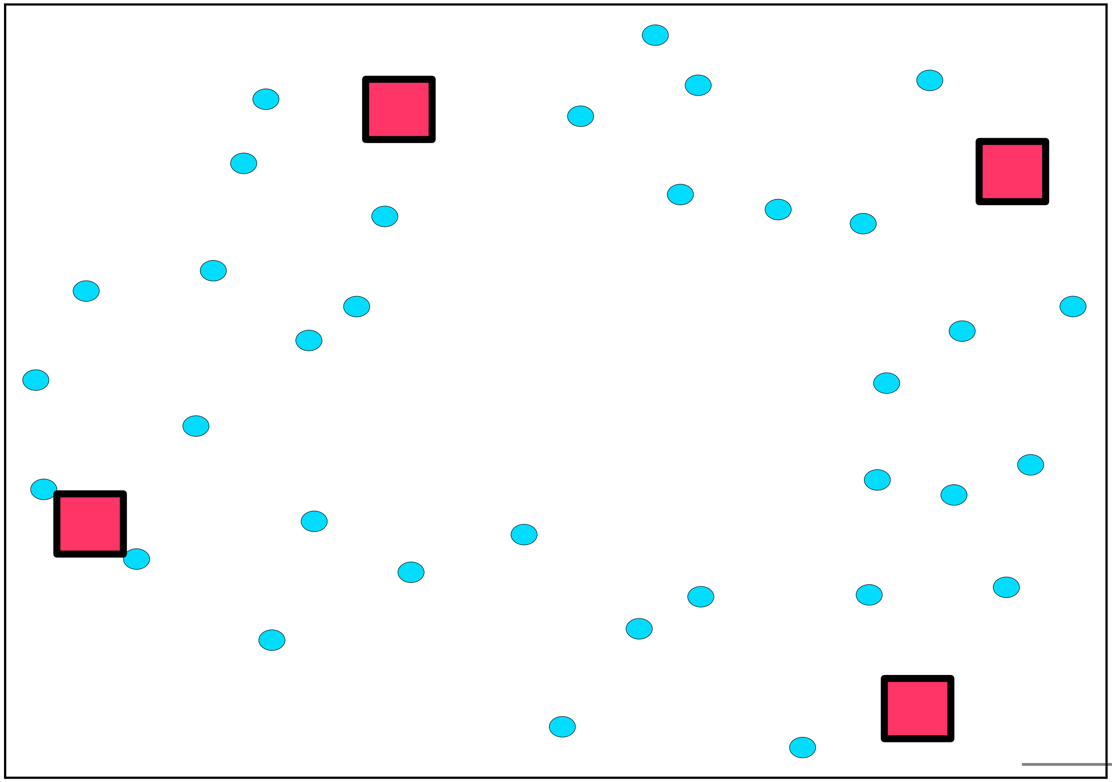
Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

- Compulsory stops



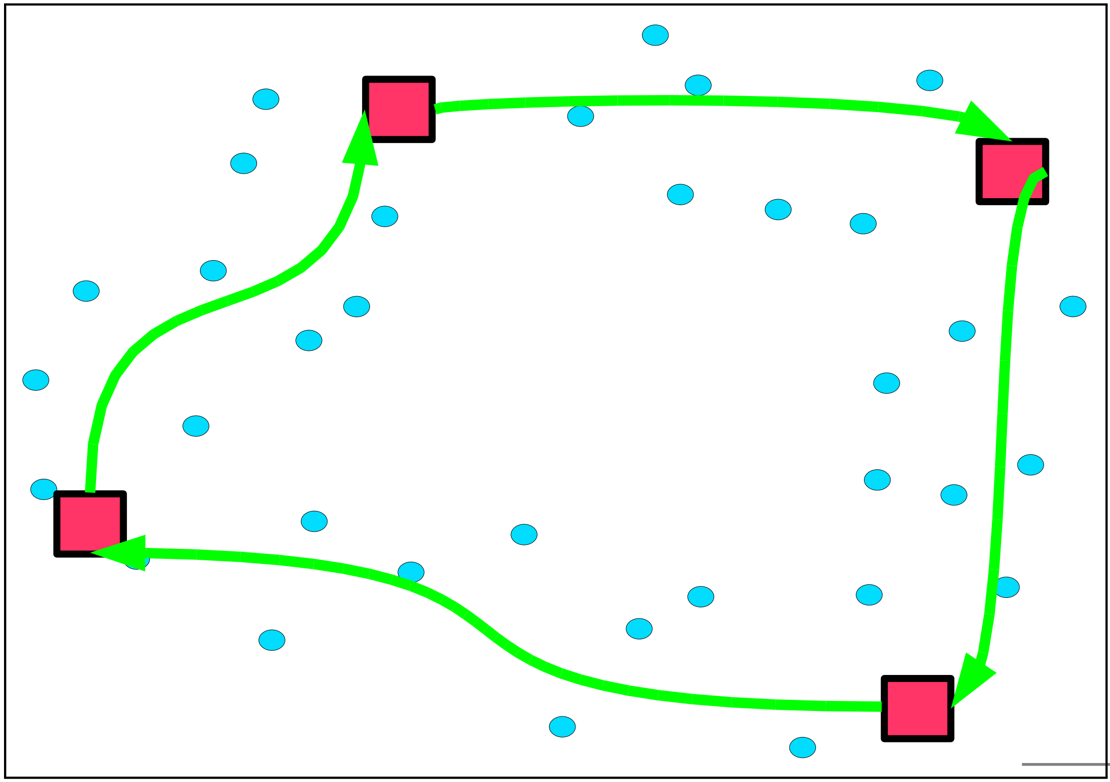
Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

- Compulsory stops
- Sequencing



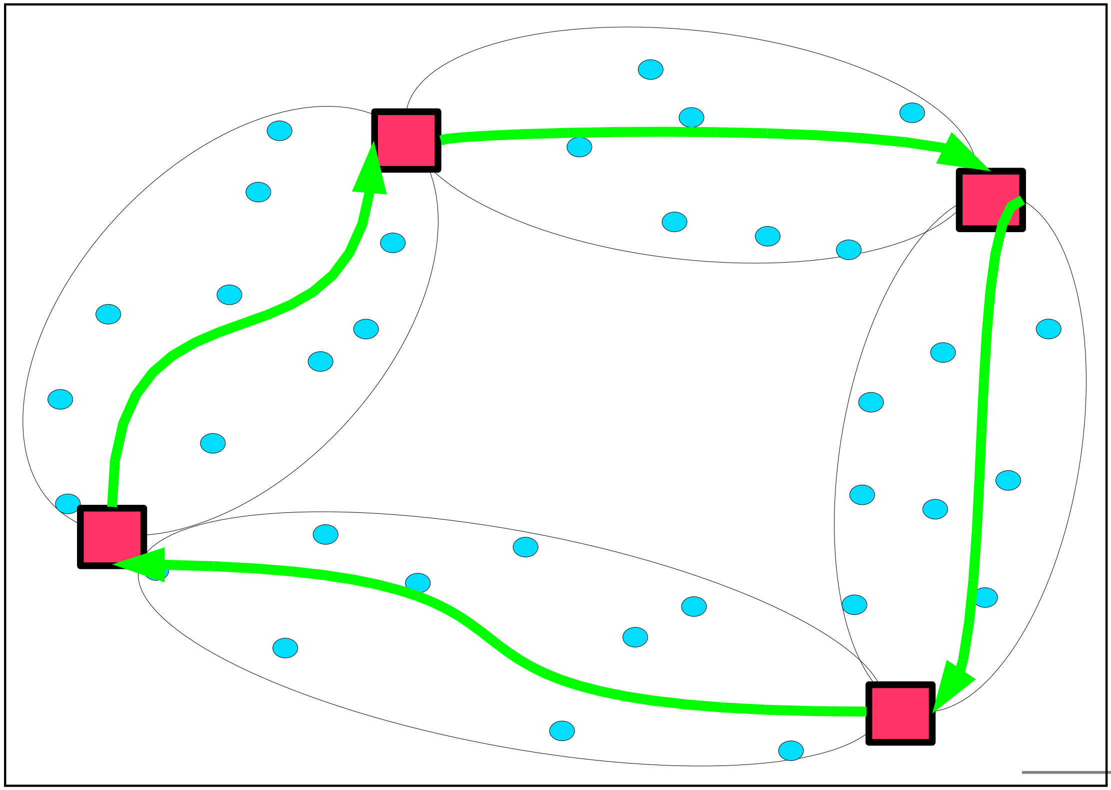
Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

- Compulsory stops
- Sequencing
- Segments



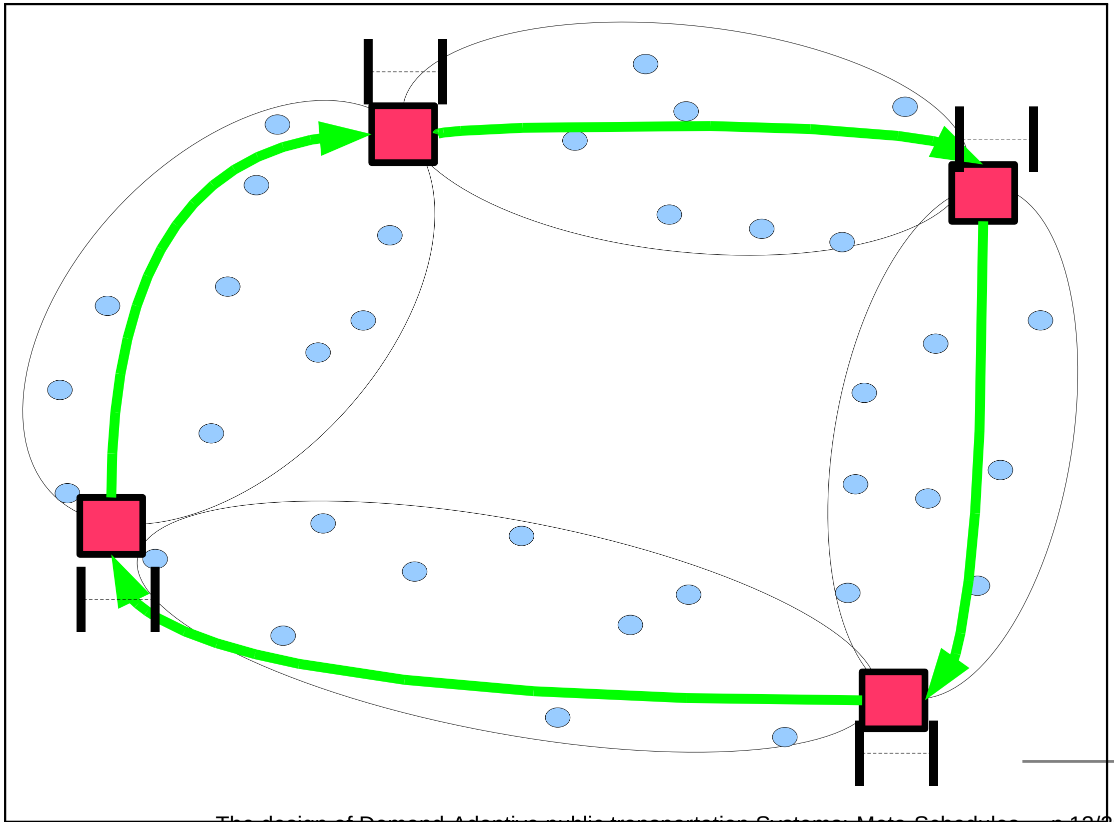
Basic DAS Design Problem

Input

- Region to serve
- Demand

Decisions

- Compulsory stops
- Sequencing
- Segments
- Time windows



Basic DAS Design Problem

Input

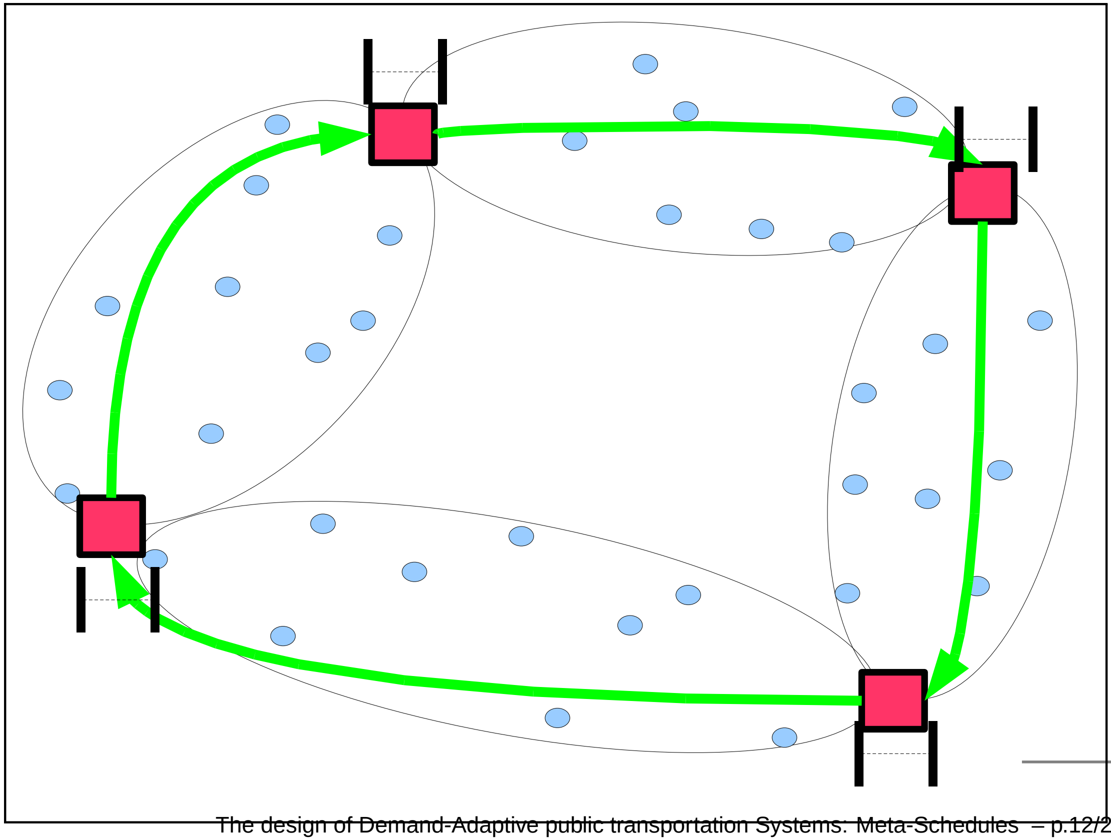
- Region to serve
- Demand

Decisions

- Compulsory stops
- Sequencing
- Segments
- Time windows

Goals

- Low routing costs
- Hight level of service



Time Windows

Time Windows

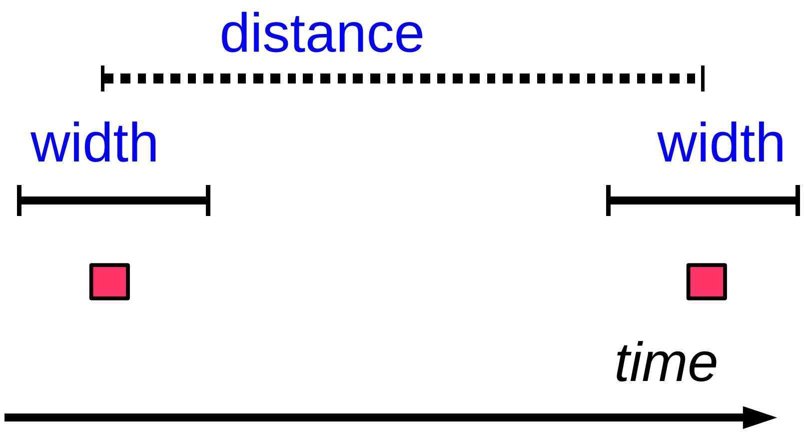
- Policy
- Given a compulsory stop and its time window $[a, b]$
- The Vehicle
  - Must leave the compulsory stop within the time interval $[a, b]$
  - May arrive *before* a

Time Windows

- Consequences
  - Passive users must be at the compulsory stop not later than a
  - Passengers on the bus may experience $idle$ times
  - Time windows represent bounds on user travel time

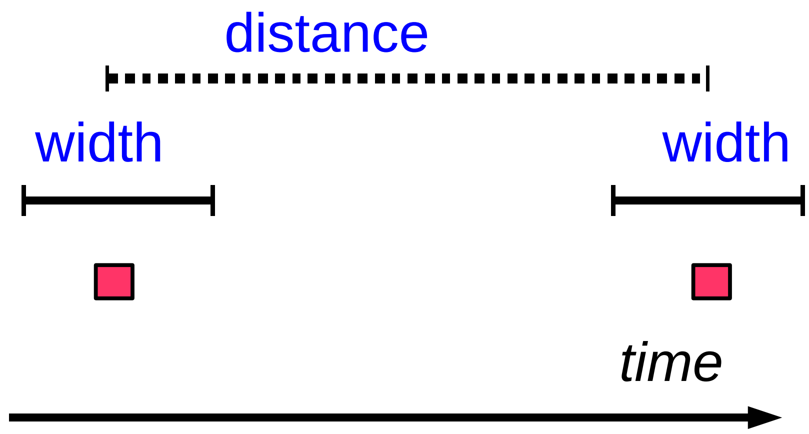
Time Windows

- Main Features
  - Distance
  - Width



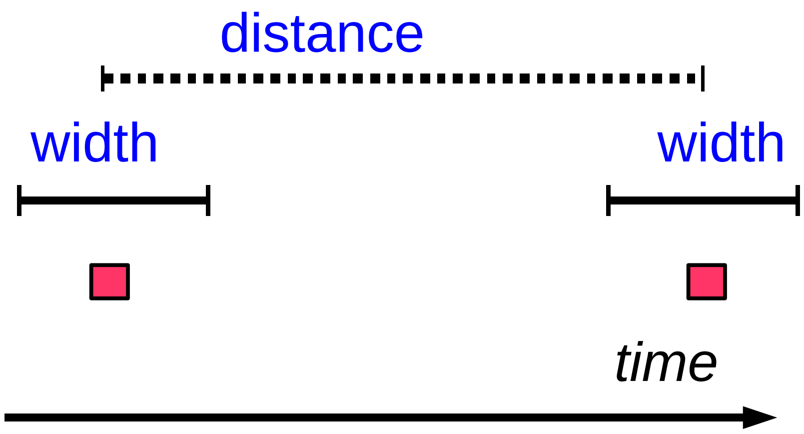
Time Windows

- Main Features
  - Distance
    - Time to serve the optional stop
    - Idle times at compulsory stops
  - Width



Time Windows

- Main Features
  - Distance
    - Time to serve the optional stop
    - Idle times at compulsory stops
  - Width
    - Flexibility
    - Long waiting times for *passive users*



Meta-Schedule: problem description

- Input
  - Topological design
  - Demand for transportation
  - Policy for time windows

Meta-Schedule: problem description

- Input
  - Topological design
  - Demand for transportation
  - Policy for time windows
- Output
  - A time window for each compulsory stop

Meta-Schedule: problem description

- Input
  - Topological design
  - Demand for transportation
  - Policy for time windows
- Output
  - A time window for each compulsory stop
- Conflicting goals
  - Provide sufficient time to serve optional stops
  - Minimize total maximum time
  - Small time windows
  - Minimize *idle* times
  - ....

Problem Setting and Assumptions

Problem Setting and Assumptions

- Demand
  - Probability distributions of requests for o/d pairs
  - $\Rightarrow$ Probability of at least one request derived for each optional stop
  - $\Rightarrow$ Serve the set of requests $\Leftrightarrow$ Serve the set of requested stops

Problem Setting and Assumptions

- Demand
  - Probability distributions of requests for o/d pairs
  - $\Rightarrow$ Probability of at least one request derived for each optional stop
  - $\Rightarrow$ Serve the set of requests $\Leftrightarrow$ Serve the set of requested stops
- Time windows
  - We consider time windows with common and fixed width δ
  - Two possible easy extensions
    - Fixed but different width
    - Maximum width

Formaly

- Input
  - A sequence of compulsory stops
  - A set of optional stops partitioned into *segments*
  - Travel time c_{ij} for the pair of stops (i, j) in a segment
  - Probability p_i of being requested for service for optional stop i

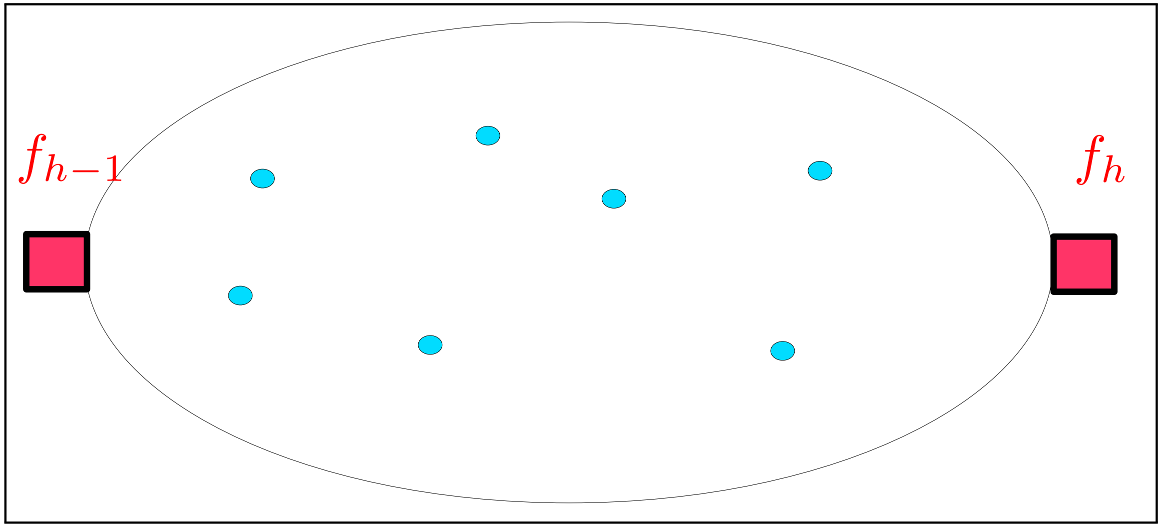
Formaly

- Input
  - A sequence of compulsory stops
  - A set of optional stops partitioned into *segments*
  - Travel time c_{ij} for the pair of stops (i, j) in a segment
  - Probability p_i of being requested for service for optional stop i
- Output
  - Time windows $[a_h, b_h]$ for compulsory stop f_h, $b_h - a_h = \delta$
  - It reduces to finding a sequence $\{b_0, b_1, \ldots, b_n\}$

Formaly

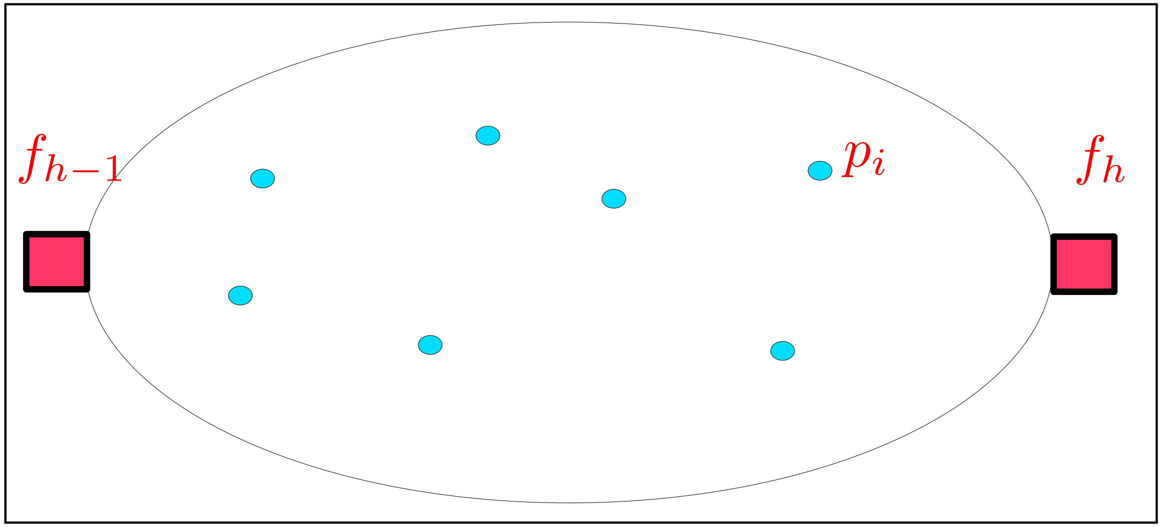
- Input
  - A sequence of compulsory stops
  - A set of optional stops partitioned into *segments*
  - Travel time c_{ij} for the pair of stops (i, j) in a segment
  - Probability p_i of being requested for service for optional stop i
- Output
  - Time windows $[a_h, b_h]$ for compulsory stop f_h, $b_h - a_h = \delta$
  - It reduces to finding a sequence $\{b_0, b_1, \ldots, b_n\}$
- Goals
  - Serve *any* requested optional stop with probability P_ϵ
  - Minimize b_n

Subproblem: Single Segment



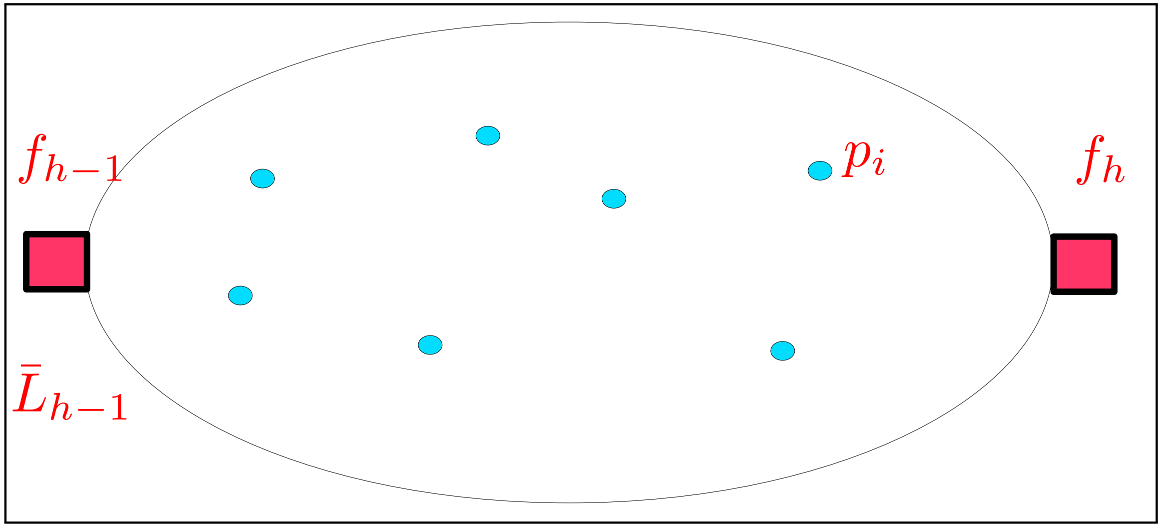
- Input
- A segment

Subproblem: Single Segment



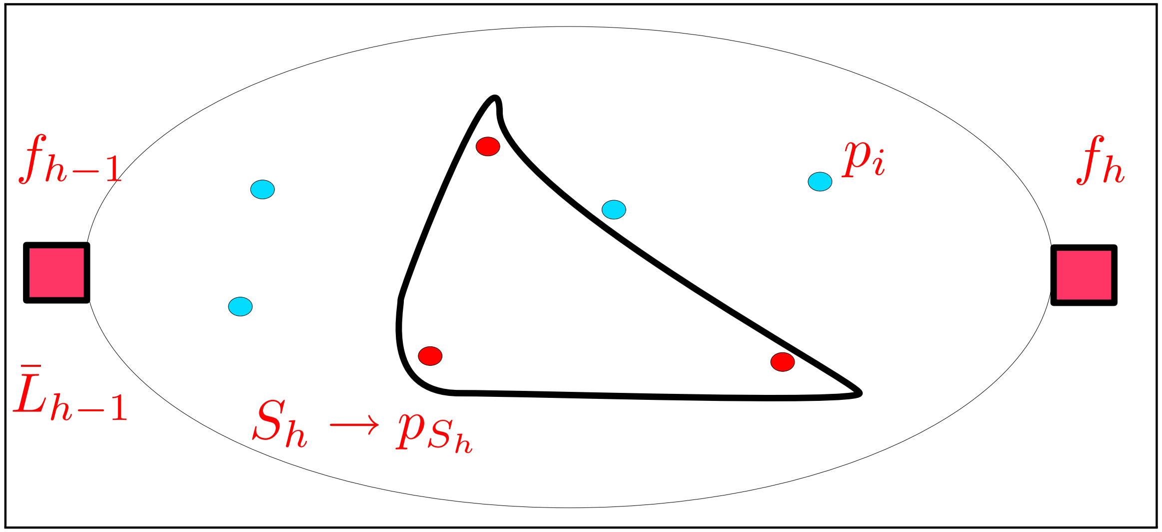
- Input
- A segment
- Probability p_i for optional stop i of being active

Subproblem: Single Segment



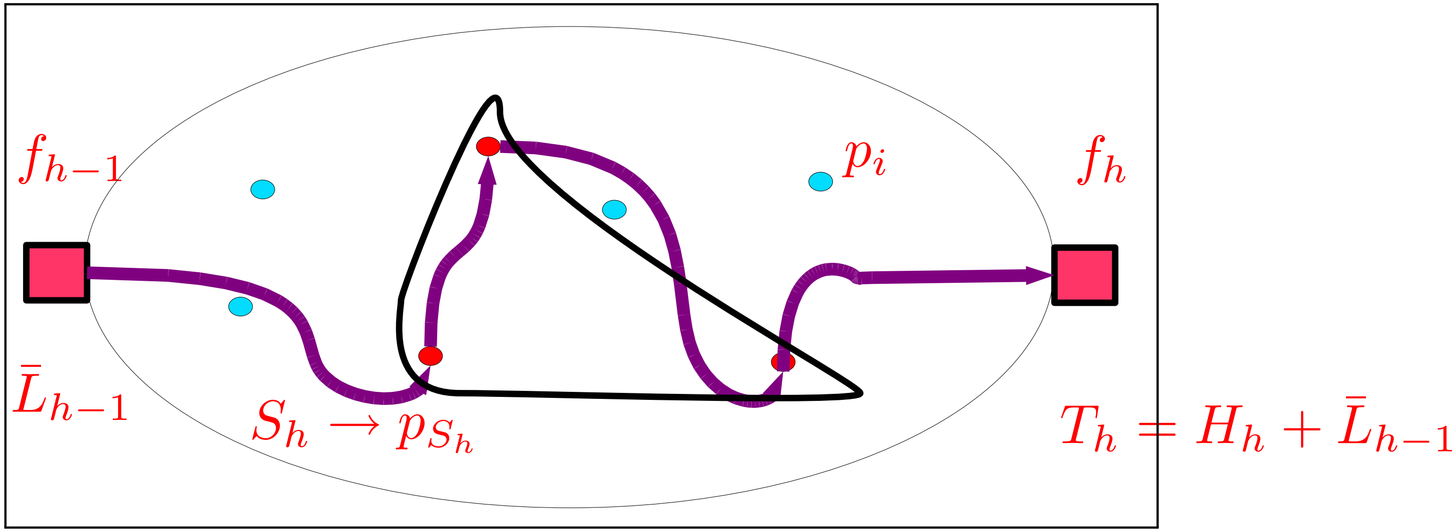
- Input
- A segment
- Probability p_i for optional stop i of being active
- The departure time $\bar{L}_{h-1}$ at compulsory f_{h-1}

Subproblem: Single Segment



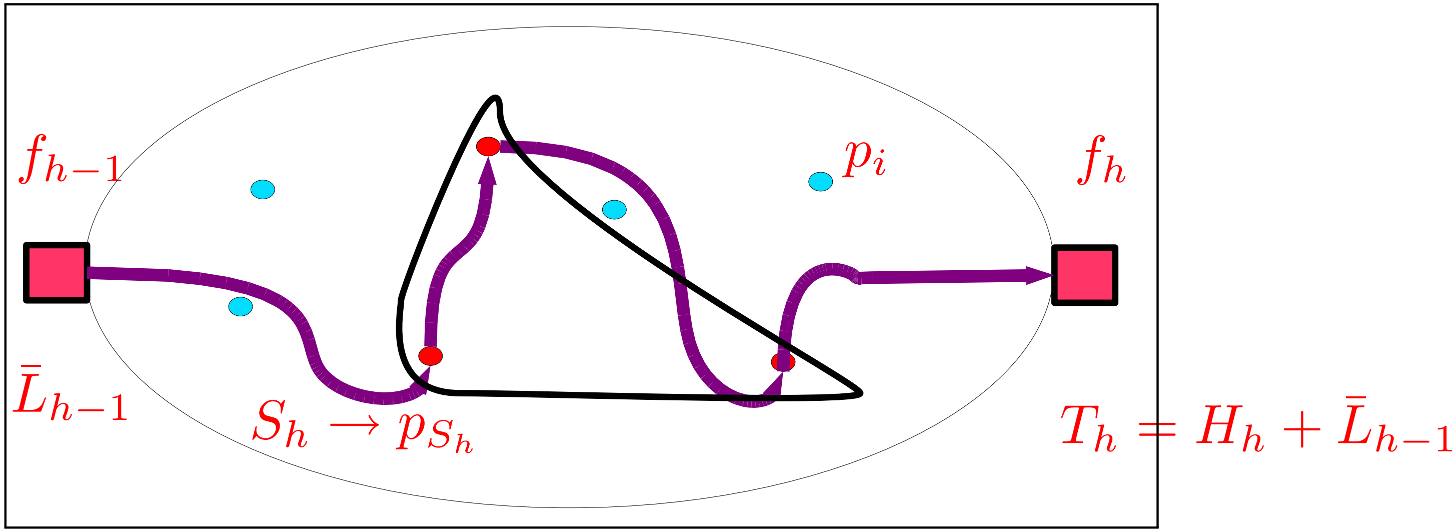
- To any subset $S_h \in N_h$ of optional stops is associated
  - Probability p_{S_h} of being active

Subproblem: Single Segment



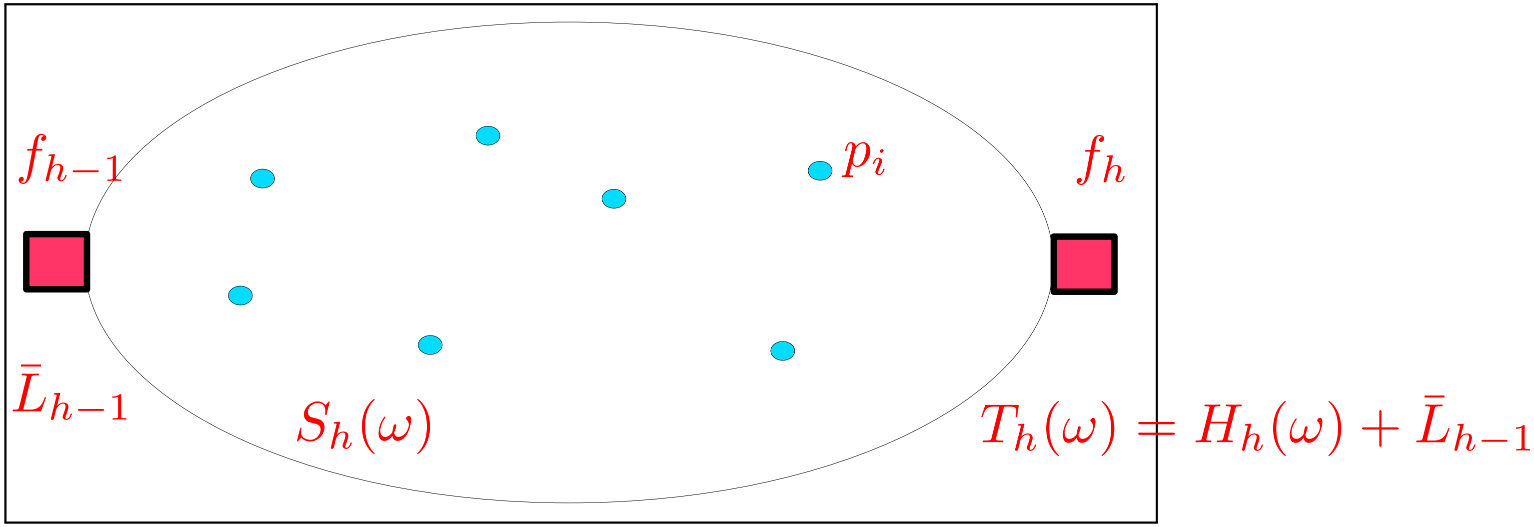
- To any subset $S_h \in N_h$ of optional stops is associated
  - Probability p_{S_h} of being active
  - Service Time $H_h \to$ (active set) Hamiltonian Path?
  - Arrival Time T_h at the second compulsory f_h

Subproblem: Single Segment



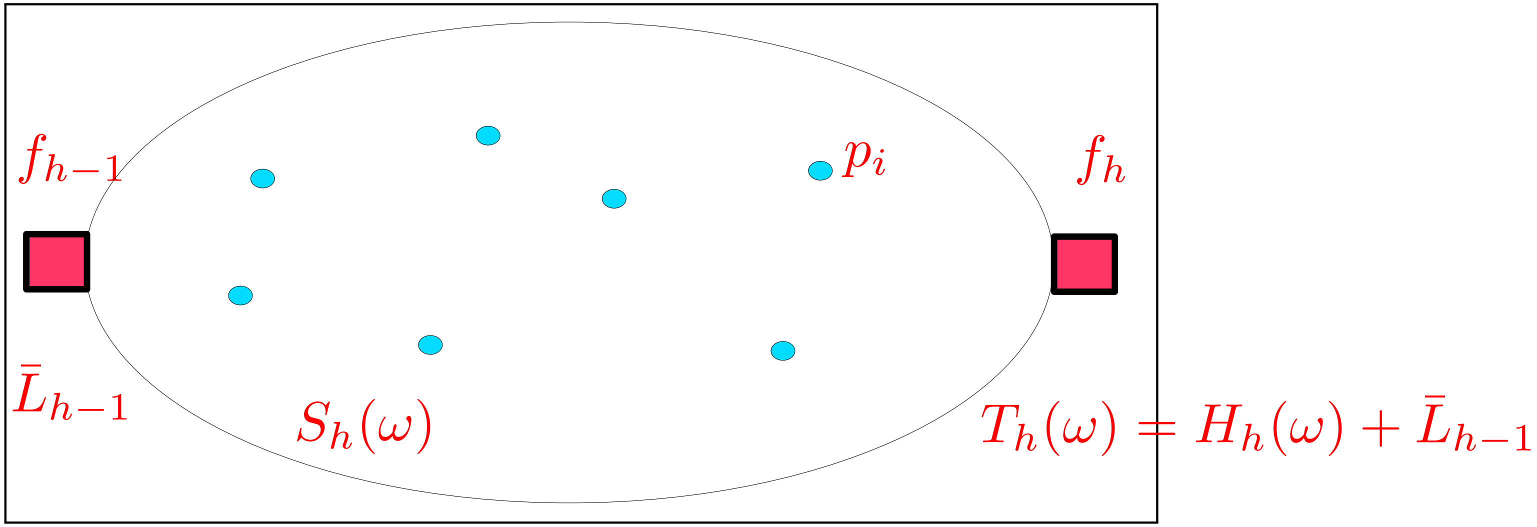
- To any subset $S_h \in N_h$ of optional stops is associated
  - Probability p_{S_h} of being active
  - Service Time $H_h \rightarrow$ (active set) Hamiltonian Path?
  - Arrival Time T_h at the second compulsory f_h
- H_h and T_h are random variables

Subproblem: Single Segment



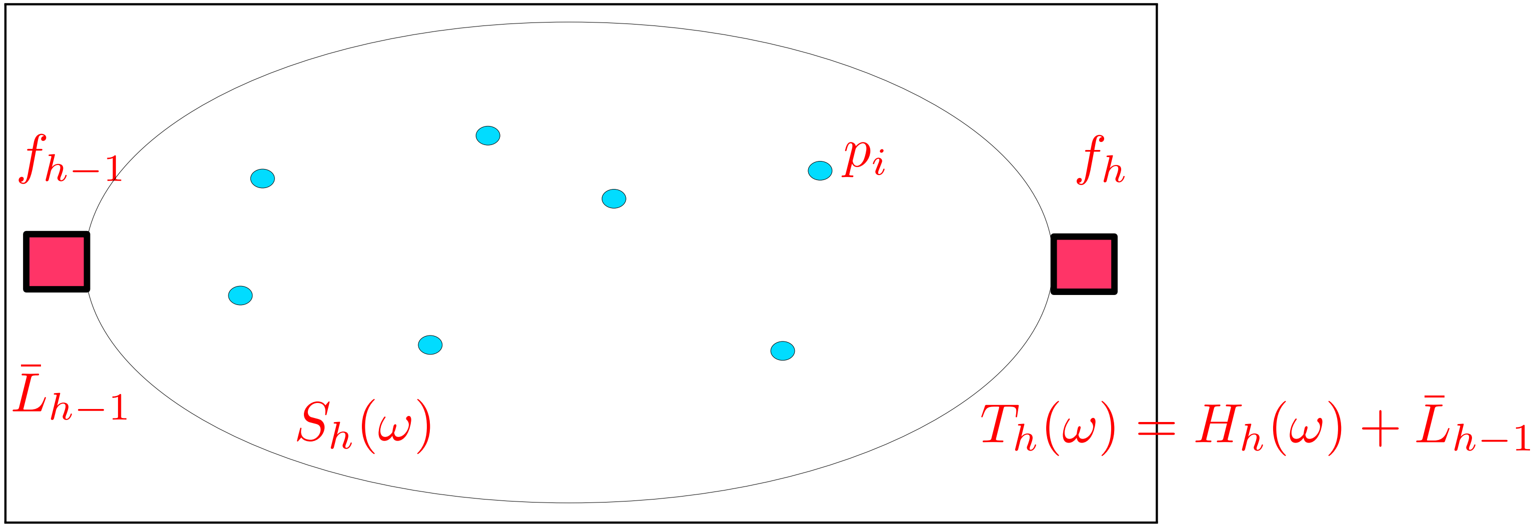
- Goal

Subproblem: Single Segment



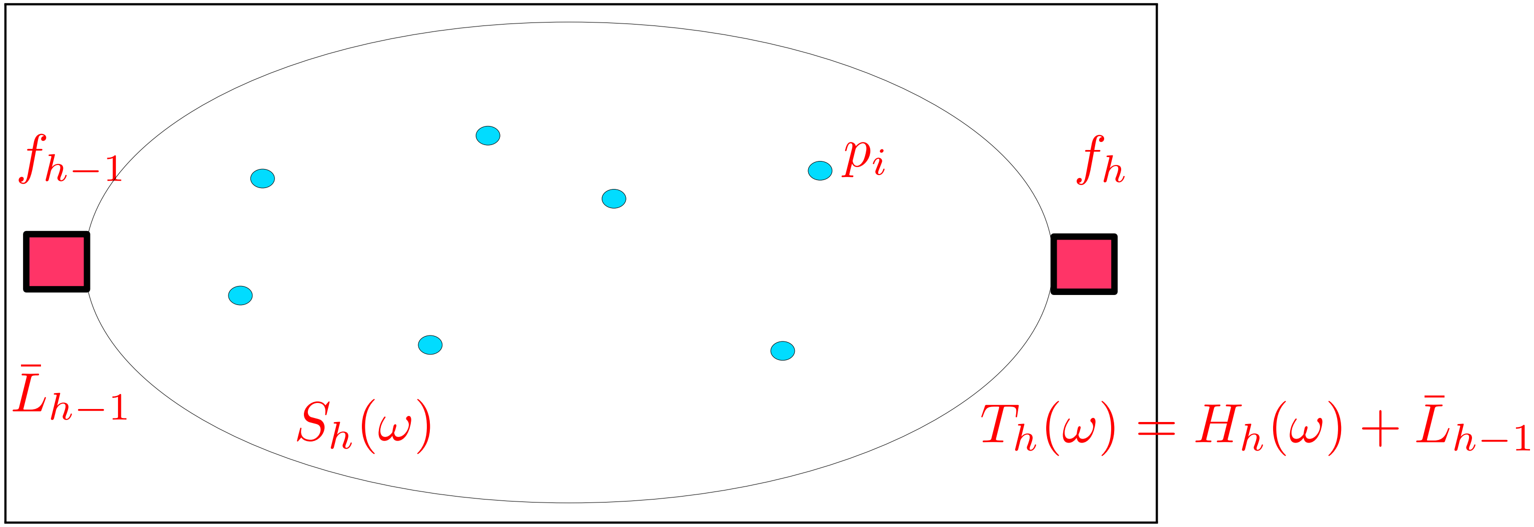
- Goal
- Find the smallest value b_h such that
- $b_h \geq T_h(\omega)$ with probability $1 - \epsilon$ (to guarantee good service)

Subproblem: Single Segment



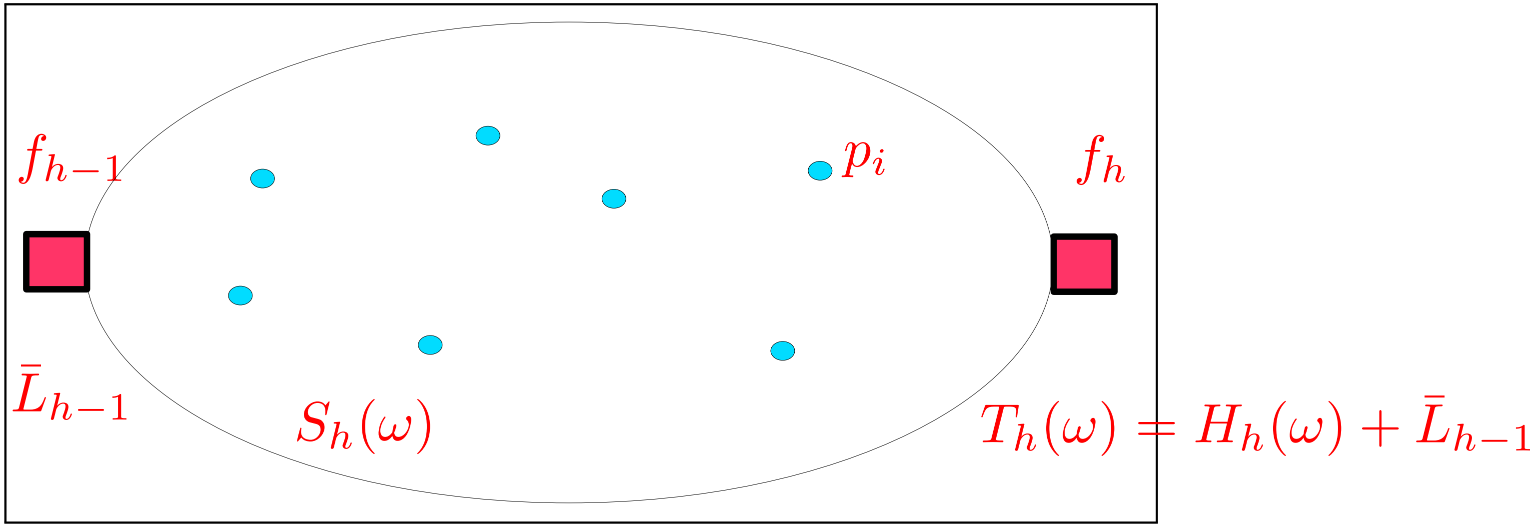
- Goal
- Find the smallest value b_h such that
- $b_h \geq T_h(\omega)$ with probability $1 - \epsilon$ (to guarantee good service)
- We take $b_h = T_h^{1-\epsilon} = H_h^{1-\epsilon} + \bar{L}_{h-1}$, with $T_h^{1-\epsilon}$, $H_h^{1-\epsilon}$ defined as
  - $\mathcal{P}\{H_h(w) \leq H_h^{1-\epsilon}\} \geq 1 - \epsilon$ and $\mathcal{P}\{T_h(w) \leq T_h^{1-\epsilon}\} \geq 1 - \epsilon$

Subproblem: Single Segment



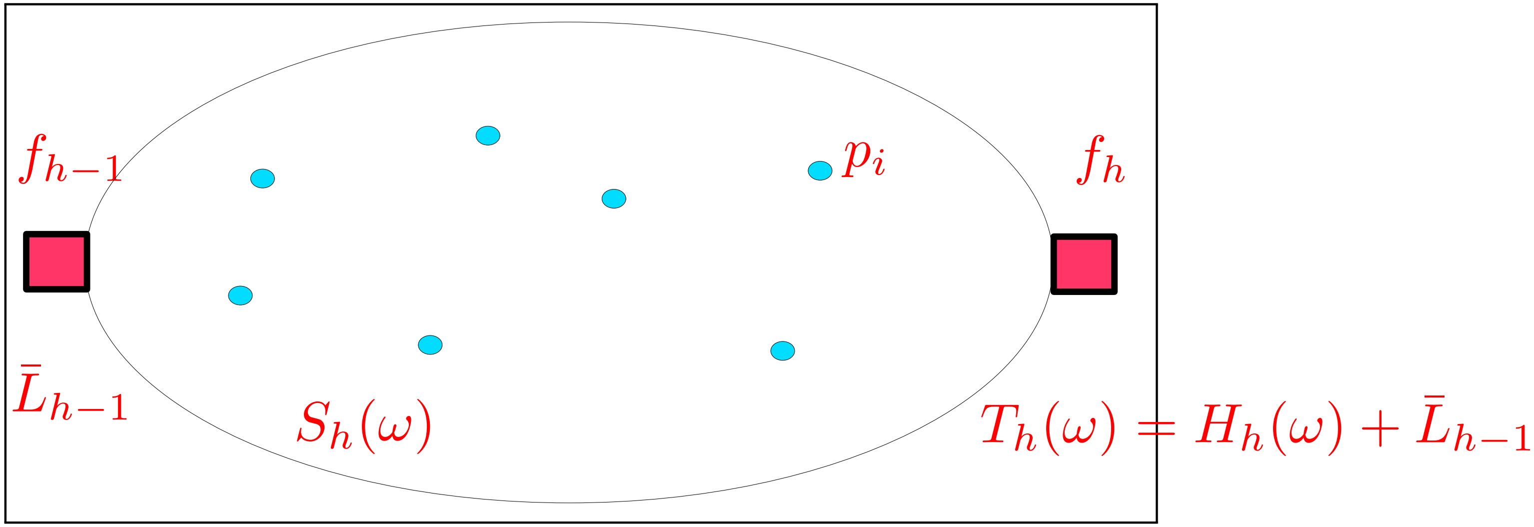
- Crucial Point

Subproblem: Single Segment



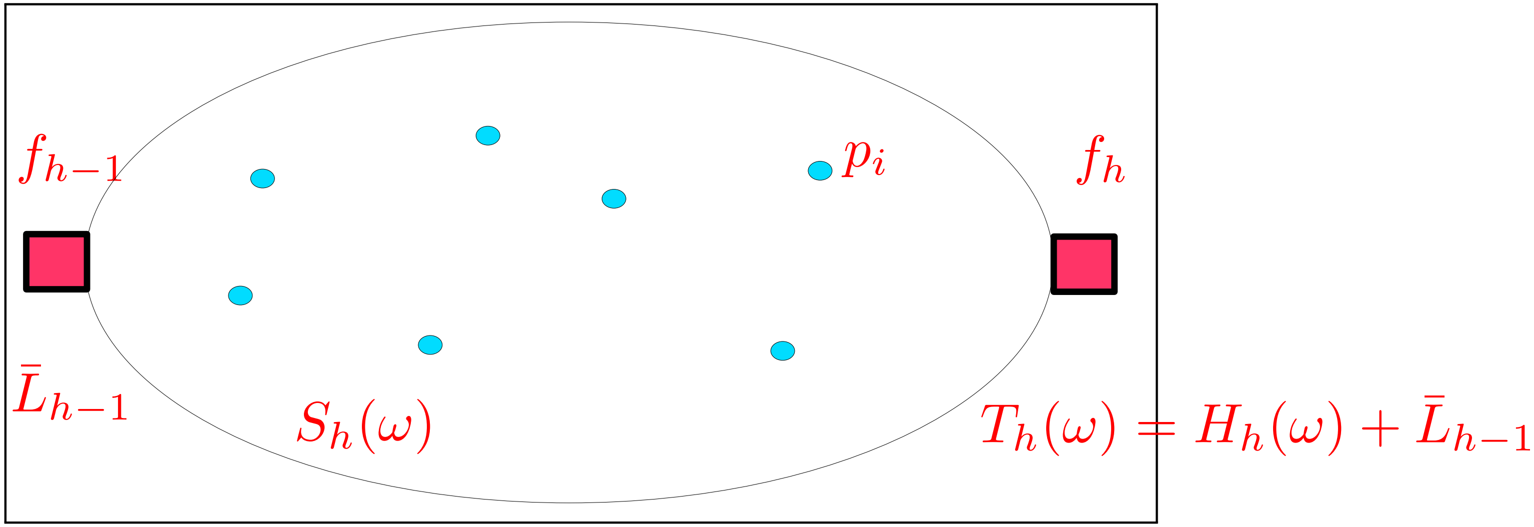
- Crucial Point
- How do we compute $T_h^{1-\epsilon}$ or $H_h^{1-\epsilon}$ (they differ by a constant)?

Subproblem: Single Segment



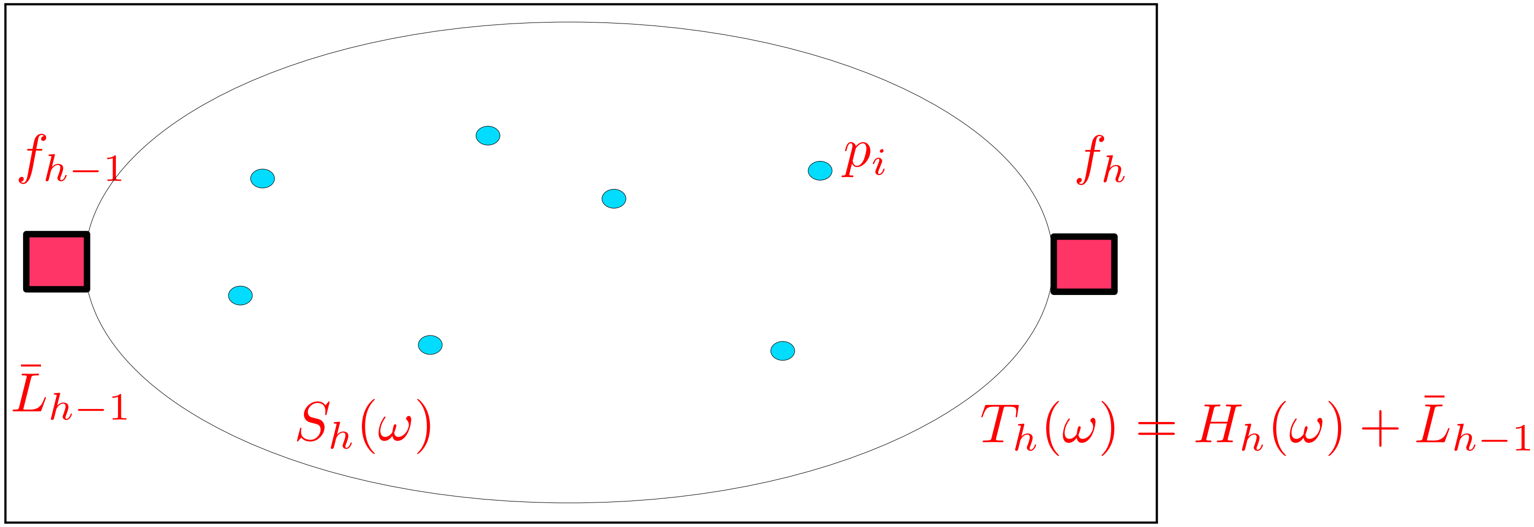
- Crucial Point
- How do we compute $T_h^{1-\epsilon}$ or $H_h^{1-\epsilon}$ (they differ by a constant)?
- Cumulative Distribution Function (CDF) and Probability Mass Function (PMF) for $H_h(\omega)$ and $T_h(\omega)$ are needed

Subproblem: Single Segment



- Crucial Point
- How do we compute $T_h^{1-\epsilon}$ or $H_h^{1-\epsilon}$ (they differ by a constant)?
- Cumulative Distribution Function (CDF) and Probability Mass Function (PMF) for $H_h(\omega)$ and $T_h(\omega)$ are needed
- Requires $2^{|N_h|}$ service-time evaluations!

Subproblem: Single Segment



- Crucial Point
- How do we compute $T_h^{1-\epsilon}$ or $H_h^{1-\epsilon}$ (they differ by a constant)?
- Cumulative Distribution Function (CDF) and Probability Mass Function (PMF) for $H_h(\omega)$ and $T_h(\omega)$ are needed
- Requires $2^{|N_h|}$ service-time evaluations!
- We *estimate* the PMF by sampling

Sampling Algorithm

- Take $\{r_1, r_2, \ldots, r_l\}$ random samples of relatively small cardinality
- Compute the PMF_k, CDF_k, and b_h^k for each sample r_k
- Compute the mean value and standard deviation of b_h^k
- If the standard deviation is small, i.e.,the solution is *precise*, stop
- Otherwise, increment the cardinality of the samples and iterate

Sampling Algorithm

- Take $\{r_1, r_2, \ldots, r_l\}$ random samples of relatively small cardinality
- Compute the PMF_k, CDF_k, and b_h^k for each sample r_k
- Compute the mean value and standard deviation of b_h^k
- If the standard deviation is small, i.e.,the solution is *precise*, stop
- Otherwise, increment the cardinality of the samples and iterate
- Observations
  - The algorithm is very simple. But
  - No guarantee of unbiased solution
  - The dimension of the sample may become critical

Experimentation

- Focus on the estimation of the service time for a single segment
- Square-shaped segments, side length 300
- Compulsory stops at the extremities of one diagonal
- Optional stops are uniformly distributed on the square
- To each optional stop is associated a positive probability, 50% in average
- $P_\epsilon = 0.95$
- Hamiltonian Path computed by our basic B&C code

Results A20

Longest 1-Path	Hamiltonian Path	“Real” $\bar{b}_h(\varepsilon)$
546.1	1137	1062.5

Dim.	nSamples	dimSamples	$\bar{b}_h(\varepsilon)$	Std-dev/$\bar{b}_h(\varepsilon)$	Time(sec.)
20	5	50	1061.5	0.012	1.9
20	5	100	1061.5	0.01	3.77
20	5	150	1060.5	0.009	5.65
20	5	200	1057.5	0.007	7.52
20	5	250	1061.5	0.008	9.35
20	10	50	1061.5	0.013	3.76
20	10	100	1056.5	0.011	7.48
20	10	150	1061	0.009	11.31
20	10	200	1062.5	0.008	15.25
20	10	250	1063	0.007	18.81
20	15	50	1061.83	0.011	5.62
20	15	100	1059.17	0.01	11.29
20	15	150	1061.17	0.01	16.86
20	15	200	1061.5	0.007	22.58
20	15	250	1062.17	0.007	28.19

Results A30

Longest 1-Path	Hamiltonian Path	“Real” $\bar{b}_h(\varepsilon)$
558.7	1244	1127.5

Dim.	nSamples	dimSamples	$\bar{b}_h(\varepsilon)$	Std-dev/$\bar{b}_h(\varepsilon)$	Time(sec.)
30	5	50	1117.5	0.011	3.74
30	5	100	1123.5	0.011	7.52
30	5	150	1124.5	0.008	11.79
30	5	200	1129.5	0.008	15.38
30	5	250	1131.5	0.008	19.41
30	10	50	1125	0.012	7.47
30	10	100	1127.5	0.013	15.4
30	10	150	1128	0.007	22.95
30	10	200	1129.5	0.006	30.13
30	10	250	1131.5	0.006	37.98
30	15	50	1123.83	0.012	11.85
30	15	100	1127.17	0.011	22.99
30	15	150	1130.17	0.007	34.58
30	15	200	1129.17	0.006	44.95
30	15	250	1130.17	0.005	55.47

Results A40

Longest 1-Path	Hamiltonian Path	“Real” $\bar{b}_h(\varepsilon)$
558.7	1444	1272

Dim.	nSamples	dimSamples	$\bar{b}_h(\varepsilon)$	Std-dev/$\bar{b}_h(\varepsilon)$	Time(sec.)
40	5	50	1271.5	0.013	11.01
40	5	100	1272.5	0.009	17.24
40	5	150	1274.5	0.008	27.9
40	5	200	1278.5	0.009	34.5
40	5	250	1277.5	0.007	42.57
40	10	50	1275	0.009	17.23
40	10	100	1279	0.012	34.47
40	10	150	1281.5	0.011	50.13
40	10	200	1278.5	0.008	63.37
40	10	250	1276.5	0.006	80.98
40	15	50	1274.17	0.011	27.85
40	15	100	1280.17	0.012	50.23
40	15	150	1278.83	0.01	74.34
40	15	200	1276.5	0.007	97.67
40	15	250	1275.5	0.005	119.65

Results A50

Longest 1-Path	Hamiltonian Path	"Real" $\bar{b}_h(\varepsilon)$
558.7	1880	1387.5

Dim.	nSamples	dimSamples	$\bar{b}_h(\varepsilon)$	Std-dev/$\bar{b}_h(\varepsilon)$	Time(sec.)
50	5	50	1387.5	0.009	33.82
50	5	100	1380.5	0.007	66.26
50	5	150	1387.5	0.01	99.03
50	5	200	1385.5	0.009	131.54
50	5	250	1387.5	0.004	164.13
50	10	50	1383.5	0.011	66.2
50	10	100	1383.5	0.01	131.49
50	10	150	1391	0.008	202.83
50	10	200	1386.5	0.007	267.47
50	10	250	1389.5	0.005	339.48
50	15	50	1382.83	0.014	98.93
50	15	100	1385.83	0.011	202.87
50	15	150	1389.5	0.007	304.37
50	15	200	1386.5	0.006	400.23
50	15	250	1388.17	0.006	494.62

Conclusions (1)

- The algorithm for estimating time windows is fast
- The number of samplings is not very important
- The cardinality of samples is important. Around 200
- Method successful in solving problems with high dimension

Sewing Segments

- We introduce a new random variable
  - The vehicle departure time $L_h(\omega)$ at compulsory stop f_h where $a_h \leq L_h(\omega) \leq b_h$
  - $T_h(\omega) = L_{h-1}(\omega) + H_h(\omega)$ needed to compute $[a_h, b_h]$
- $T_h(\omega)$ and $L_h(\omega)$ are linked by the relation

$$L_h(\omega) = \begin{cases} T_h(\omega) & \text{if } \omega \mid a_h \leq T_h(\omega) \leq b_h; \\ a_h & \text{if } \omega \mid T_h(\omega) < a_h; \\ b_h & \text{if } \omega \mid T_h(\omega) > b_h. \end{cases}$$

- Observe that $L_{h-1}(\omega)$ and $H_h(\omega)$ are independent
- $T_h(\omega)$ can be computed by convoluting $L_{h-1}(\omega)$ and $H_h(\omega)$

Complete Scheme

- Input
  - Sequence of segments $G = \cup_{1,2,\ldots,n} G_h$
  - Service probability $P_\epsilon = (1-\epsilon)^n$

Complete Scheme

- Input
  - Sequence of segments $G = \cup_{1,2,\dots,n} G_h$
  - Service probability $P_\epsilon = (1-\epsilon)^n$
- Algorithm
  1. For each segment $G_h,\ h \in \{1, 2, \dots, n\}$
     (a) Compute PMF and CDF of $L_{h-1}(\omega)$
     (b) Compute PMF and CDF of $H_h(\omega)$
     (c) Compute PMF and CDF of $T_h(\omega)$ as the convolution of the PMFs of L_{h-1} and H_h
     (d) Compute $T_h^{1-\epsilon}$ and set $b_h = T_h^{1-\epsilon}$.

Complete Scheme

- Input
  - Sequence of segments $G = \cup_{1,2,\ldots,n} G_h$
  - Service probability $P_\epsilon = (1-\epsilon)^n$
- Algorithm
  1. For each segment $G_h,\ h \in \{1, 2, \ldots, n\}$
     (a) Compute PMF and CDF of $L_{h-1}(\omega)$
     (b) Compute PMF and CDF of $H_h(\omega)$
     (c) Compute PMF and CDF of $T_h(\omega)$ as the convolution of the PMFs of L_{h-1} and H_h
     (d) Compute $T_h^{1-\epsilon}$ and set $b_h = T_h^{1-\epsilon}$.
- Output
  - The best sequence $\{b_1, b_2, \ldots, b_n\}$
  - Such that any randomly requested optional stop is served with probability P_ϵ

Experimentation

- Goal
  - evaluate the quality of the produced master schedules
- Given several:
  - topological DAS line design
  - origin destination demand distributions
- Compute several master schedules varying:
  - time window width δ
  - probabilities ϵ
- Simulate operations monitoring:
  - percentage of rejected requests
  - passive user waiting times
  - idle times at compulsory
  - ...

Results

ϵ	δ	b_4	avg(R/T)	stdv(R/T)	avg(I)	stdv(I)	avg(P)	stdv(P)
0.90	300	3032	0.008	0.028	68.09	54.64	46.17	43.76
0.90	400	3017	0.006	0.023	45.14	52.04	88.97	74.86
0.90	500	3117	0.006	0.023	30.30	44.21	148.70	108.48
0.95	300	3152	0.004	0.020	92.47	62.20	35.25	42.70
0.95	400	3132	0.004	0.020	65.47	54.62	61.40	64.54
0.95	500	3132	0.004	0.019	46.70	53.37	108.09	93.49
0.90	300	3692	0.016	0.021	9.1	26.30	154.1	71.20
0.90	400	3692	0.014	0.020	4.7	20.12	243.0	85.83
0.90	500	3692	0.015	0.021	2.7	15.11	345.79	96.03
0.95	300	4022	0.009	0.017	11.7	28.36	137.51	70.40
0.95	400	4022	0.011	0.018	6.6	23.46	222.95	89.44
0.95	500	4022	0.011	0.016	3.5	17.85	312.75	98.98

Master schedule evaluated on 4-segment line, low and high demand, 100 realizations

Conclusions (2)

- Demand Adaptive transit Systems
- DAS scheduling different from DAR and traditional transit
- The Meta-Schedule problem
- May be efficiently addressed by sampling (single segment)
- Results for general framework are satisfactory under different demand and line scenarios