# Scheduled Service Network Design for Rail Carriers

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# Outline

1. Freight Rail Transportation
2. Service Network Design Problem
3. 2-Layer Time-Space Network
4. Mathematical Formulation
   - Variables
   - Objective
   - Constraints
5. Future Works

# Freight Rail Transportation

- Rail transportation is a tremendous industry, and one of the most substantial transportation ways for modern society,
  - Carried 269.8 million tons of freight in Canada (2004).[1]
  - Achieved revenue nearly $7.9 billion in Canada (2004).[1]
- Very rich in terms of the planning and scheduling problems.

[1] Statistics Canada

# Complicated Freight Rail Operations

- Cars are classified and formed into blocks in classification yards.
- A block is a group of cars with different origins and destinations, but will be transported together from block origin until the block destination.
- Trains transport blocks on rail network.
- At an intermediate stop, blocks may be unloaded and transferred to another train.
- From the origin to the destination, each car may go through one or several blocks and trains.

# Service Network Design Problem

The *service network design* focuses on generating a good operating plan to satisfy the transportation demands, while operating in a smooth, rational, and cost-efficient way.

- Service Selection
- Blocking Policy
- Makeup Policy
- Traffic Distribution

# Service Network Design Problem

The *service network design* focuses on generating a good operating plan to satisfy the transportation demands, while operating in a smooth, rational, and cost-efficient way.

- Service Selection
  - on which route should we provide the train service?
  - at what speed should the train runs?
  - when should we provide the train service?
  - what's the arrival/departure time in each intermediate stop?
- Blocking Policy
- Makeup Policy
- Traffic Distribution

# Service Network Design Problem

The *service network design* focuses on generating a good operating plan to satisfy the transportation demands, while operating in a smooth, rational, and cost-efficient way.

- Service Selection
- Blocking Policy
    - what blocks should be built at each yard?
    - which cars enter which block?
- Makeup Policy
- Traffic Distribution

# Service Network Design Problem

The *service network design* focuses on generating a good operating plan to satisfy the transportation demands, while operating in a smooth, rational, and cost-efficient way.

- Service Selection
- Blocking Policy
- Makeup Policy
    - which train takes which block?
- Traffic Distribution

# Service Network Design Problem

The *service network design* focuses on generating a good operating plan to satisfy the transportation demands, while operating in a smooth, rational, and cost-efficient way.

- Service Selection
- Blocking Policy
- Makeup Policy
- Traffic Distribution
  - on which route should each commodity be sent?
  - what operations should be performed at each intermediary stop?

# Previous Models

**Blocking Problem** Bodin *et al.* (1980), Newton *et al.* (1998), Barnhart *et al.*(2000), Ahuja *et al.* (2004), etc.

**Train Routing w/ Frequency** Assad (1980), Van Dyke (1986, 1988), Keaton (1989, 1992), Marín and Salmerón (1996), etc.

**Train Routing w/ Schedule** Huntley *et al.*(1995), Newman and Yano (2000), etc.

**Compound Models** Crainic *et al.* (1984), Haghani (1989), Nozick and Morlok (1997), Gorman (1998), etc.

**Recent Reviews** Cordeau *et al.* (1998), Crainic (2000), Newman *et al.* (2002).

# Main Goal

The research aims to generate a good operating plan for rail carriers.

Given the demand pattern and rail character, we would like to generate,

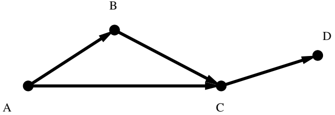
- Scheduled (Time-Dependent) Train Service Plan
- Blocking Policy
- Makeup Policy
- Traffic Itinerary Design

# Project

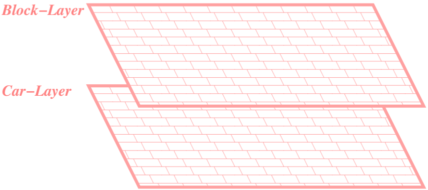
By considering the issues simultaneously and analyzing the trade-offs among them, our problem is pretty complex.

| | |
|---|---|
| Simplification | direct (non-stop) services. |
| Main Goal | complete services with intermediate stops. |
| Intensification | non-linear model. |

# 2-Layer Time-Space Structure

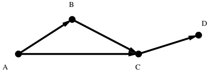



PHYSICAL NETWORK




2−LAYER TIME−SPACE NETWORK

# Block Layer

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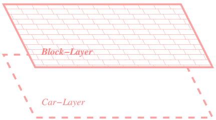
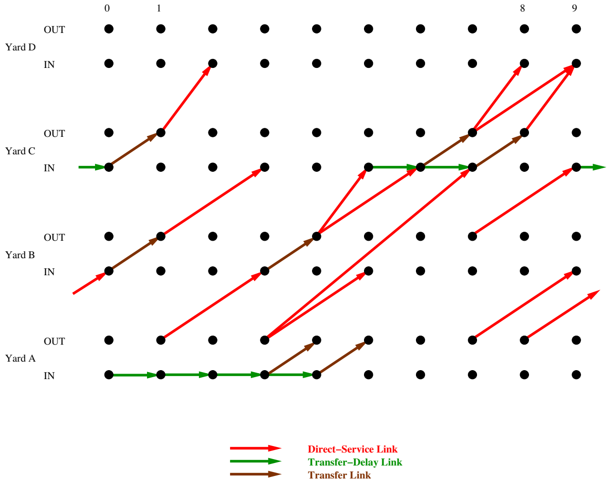



TIME−SPACE NETWORK



# Services

Each service is represented by a *direct-service link* in block layer.

For each service $s \in S$,

- specific departure time, and fixed service time;
- flow capacity $u_s$ in number of cars;
- linear flow cost for each car;
- fixed cost representing the locomotive and crew cost;
- no intermediate stops.

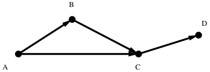
# Blocks

To further describe the decision on blocking policy, we need to build blocks.

Each block *b* can be represented by a "path" in block-layer,

- the "path" is formed by a series of direct-service links, which are connected by transfer-delay links and transfer links;
- flow cost is the sum of the flow costs on links;
- an approximated occupancy time for one classification track in origin yard;
- fixed cost representing the classification track occupancy cost in origin yard.

# Blocks

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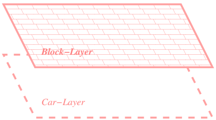
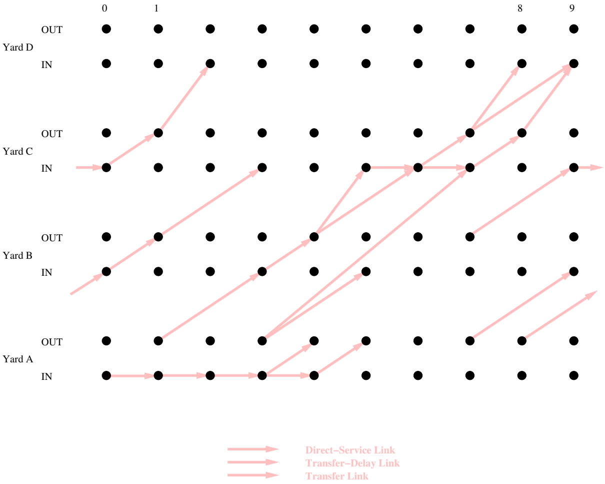
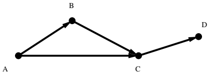



TIME–SPACE NETWORK



# Blocks

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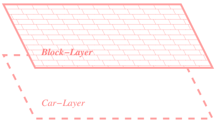
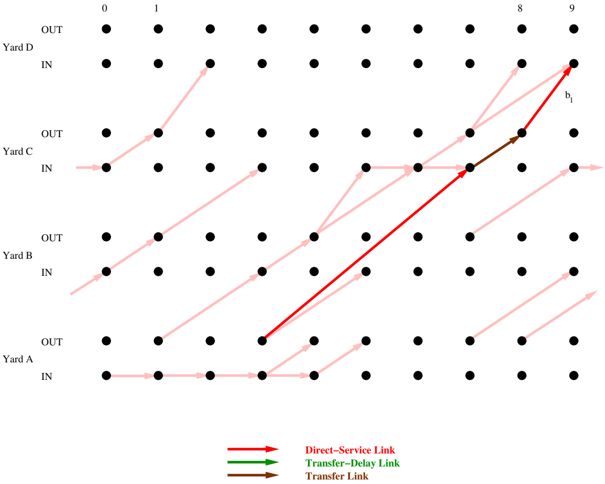



TIME−SPACE NETWORK



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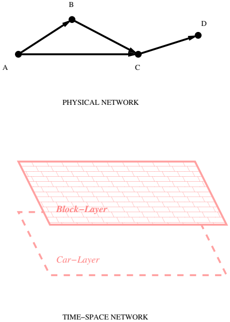
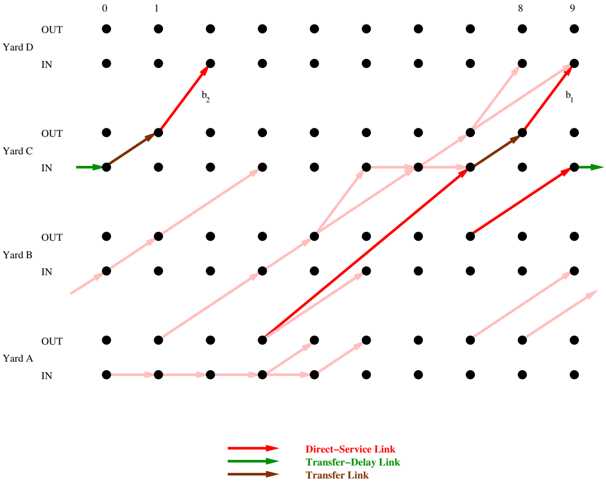
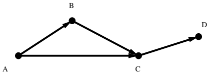

B
D
A
C
PHYSICAL NETWORK
Block−Layer
Car−Layer
TIME−SPACE NETWORK


0
1
8
9
OUT
Yard D
IN
$b_2$
$b_1$
OUT
Yard C
IN
OUT
Yard B
IN
OUT
Yard A
IN
Direct−Service Link
Transfer−Delay Link
Transfer Link

# Blocks

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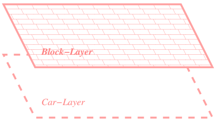


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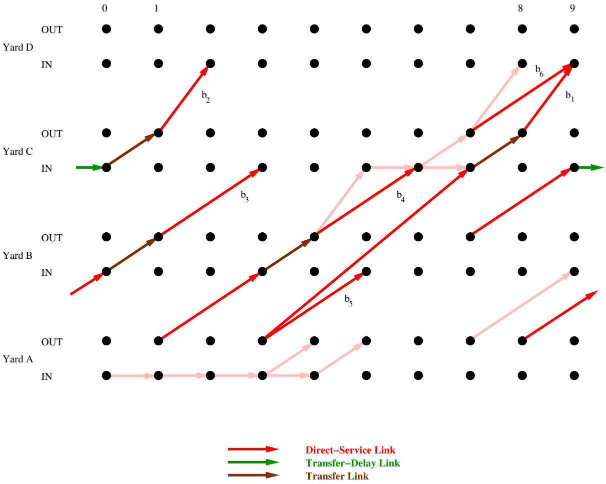
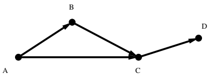


# Car-Layer

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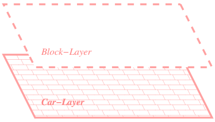


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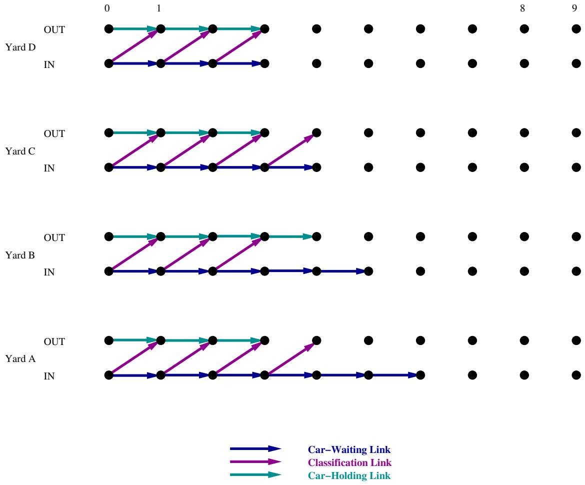
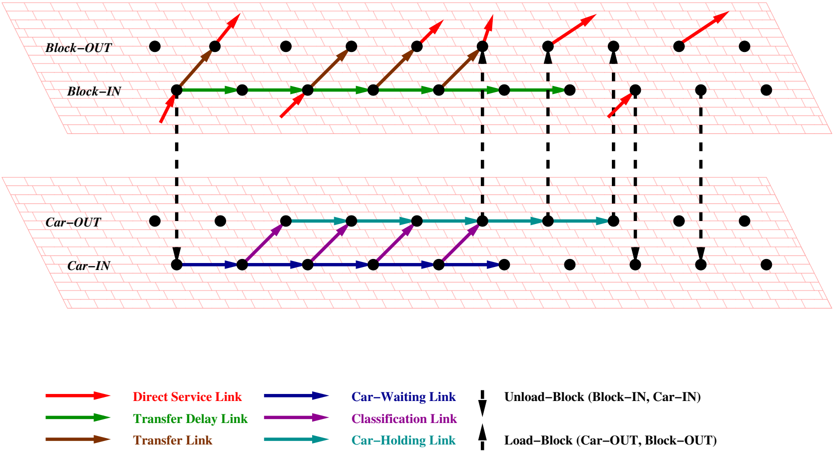
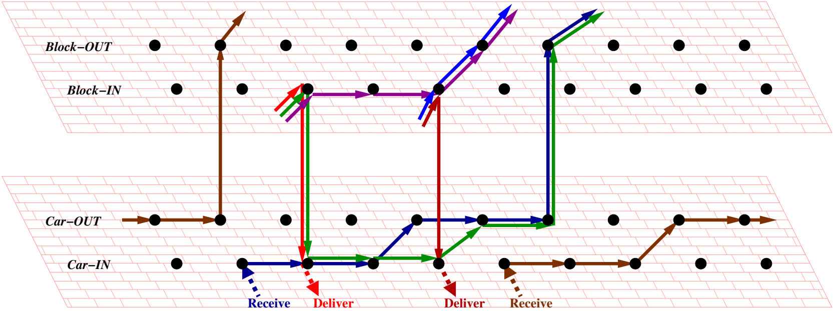


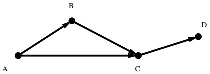
# Inter-Layer Links



# Flows in Time-Space Network



# Flows in Time-Space Network (Vertical Projection)

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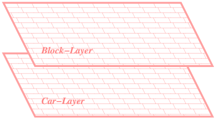


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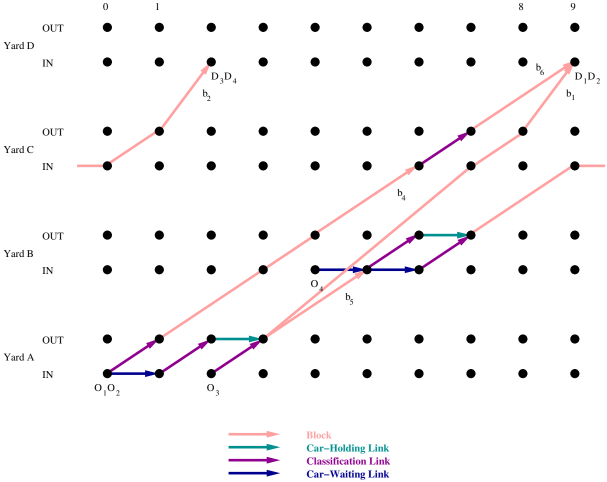


# Traffic Class

$p = (O, D, C, r, d)$

- $O$ - origin terminal;
- $D$ - destination terminal;
- $C$ - type of commodity;
- $r$ - receiving time point. The time when cars are available for shipping;
- $d$ - due transit time. From $r(p)$, by the requirement from customer, traffic $p$ must arrive at destination in $d(p)$ time periods.

# Variables

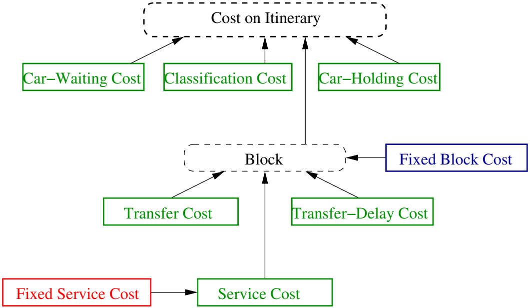
$x_a^p \geq 0$ number of cars of traffic class $p$ traveling on a link $a \in A$.

$y_b \in \{0, 1\}$ if we build block $b \in B$, $y_b = 1$; otherwise, $y_b = 0$.

$z_s \in \{0, 1\}$ if we provide service $s \in S$, $z_s = 1$; otherwise, $z_s = 0$.

# Objective

By ensuring the proper delivery for customers, the objective is minimizing the total operating cost generated.



# Constraints

- Conservation Constraint.
  - Car Flow Conservation
- Forcing Constraints.
  - Car Handling Capacity — Each yard has a capacity on the number of cars that can be classified in each time period.
  - Train Length Capacity — Each train can haul limited number of cars.
  - Block Building Capacity — Number of blocks being built in one yard is constrained by the number of classification tracks.
  - Train Running Capacity — Constraint on the number of trains running on a physical rail track section, depending on the physical condition and track mile, etc.
- Linking Constraints
  - Services <=> Blocks
  - Blocks <=> Cars

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  - Blocks <=> Cars

# Formulation

$$\min \Phi = \sum_{p \in P} \sum_{a \in A} c(p,a) \cdot x_a^p + \sum_{b \in B} c^f(b) \cdot y_b + \sum_{s \in S} c^f(s) \cdot z_s \tag{1}$$

$$\text{s.t.} \sum_{a \in A^+(n)} x_a^p - \sum_{a \in A^-(n)} x_a^p = w_n^p \qquad \forall n \in N, \forall p \in P; \tag{2}$$

$$\sum_{p \in P} x_a^p \leq u_a \qquad \forall a \in A^c; \tag{3}$$

$$\sum_{p \in P} \sum_{b \in B | s \in S(b)} x_b^p \leq z_s u_s \qquad \forall s \in S; \tag{4}$$

$$\sum_{b \in B(a)} y_b \leq u_{v(a)} \qquad \forall a \in A^h; \tag{5}$$

$$\sum_{s \in S(e,t)} z_s \leq u_e \qquad \forall e \in E, \forall t \in \{1, \cdots, \mathbf{T}\}; \tag{6}$$

$$\sum_{p \in P} x_b^p \leq y_b u_b \qquad \forall b \in B; \tag{7}$$

$$\sum_{b \in B | s \in S(b)} y_b \leq z_s u_s \qquad \forall s \in S. \tag{8}$$

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# Future Works

1. Solution Methods
   - Meta-Heuristics.
   - Branch & Price & Cut.
2. General Model
   - Complete Services - with intermediary stops.
   - Complex Time-Space Network with more layers.
3. Address the Non-Linear Phenomenons
   - In-Yard Operations.
   - On-Track Transportation.

Thanks for your attention.

Joyeux Noël!

# Notation

| | |
|---|---|
| $G$ | $(V, E)$ the physical network. |
| $P$ | set of traffic classes. |
| T | maximal time periods. |
| $N$ | set of nodes in each layer. |
| $A^i$ | set of inter-service transfer links. |
| $A^d$ | set of transfer-delay links. |
| $S$ | set of direct-service links. |
| $A(S)$ | $A^i \cup A^d \cup S$ link set of block-layer. |
| $A^w$ | set of car-waiting links. |
| $A^c$ | set of classification links. |
| $A^h$ | set of car-holding links. |
| $B$ | set of blocks. |
| $A$ | $A^w \cup A^c \cup A^h \cup B$ link set of car-layer. |

# Notation

| | |
|---|---|
| $c(p,a)$ | linear cost for shipping one car of traffic class $p$ on link $a \in A$. |
| $c^f(b)$ | fixed cost for opening block $b \in B$. |
| $c^f(s)$ | fixed cost for opening one direct service $s \in S$. |
| $B(a)$ | set of blocks that start at the yard car-holding link $a$ represents, and for each $b \in B(a)$, $a$ is in the $\tau_b$ time periods before $O(b)$. |
| $S(e,t)$ | set of direct-service links that represent train running on physical link $e \in E$ in time period $t \in \{1, \cdots, \mathrm{T}\}$. |
| $S(b)$ | set of direct service links that used by block $b$. |
| $u(a)$ | the temporal length of link $a \in A$. |
| $x_a^p$ | $\geq 0$, the flow of traffic $p$ on $a \in A$. |
| $y_b$ | $\in \{0, 1\}$, decision on building the block $b \in B$. |
| $z_s$ | $\in \{0, 1\}$, decision on providing the direct-service $s \in S$. |

# Notation

| | |
|---|---|
| $d^p$ | demand for traffic class $p \in P$. |
| $A^+(n)$ | set of links $a \in A$ that depart from node $n$. |
| $A^-(n)$ | set of links $a \in A$ that end at node $n$. |
| $w_n^p$ | if $n$ is the origin of traffic $p$, $w_n^p = d^p$; if $n$ is the destination of traffic $p$, $w_n^p = -d^p$; otherwise $w_n^p = 0$. |
| $u_a$ | maximum of cars can be handled on classification link $a$. |
| $u_s$ | maximum of cars that a train can take on direct-service link $s$. |
| $u_b$ | maximum of cars that block $b$ can take. |
| $u_{v(a)}$ | maximal number of blocks can be built at the same time in yard $v \in V$, which is represented by car-holding link $a$. |
| $u_e$ | maximal number of train running on physical track $e$ at the same time. |